Microwave Circuits and Antenna Design

Two-Port Amplifier Design Using S-parameters

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Amplifier Design using S-parameters

- Two-Port Power Gain
- Stability
- Unilateral Gain
- CONSTANT GAIN CIRCLES

Two-Port Power Gain

TWO-PORT POWER GAIN

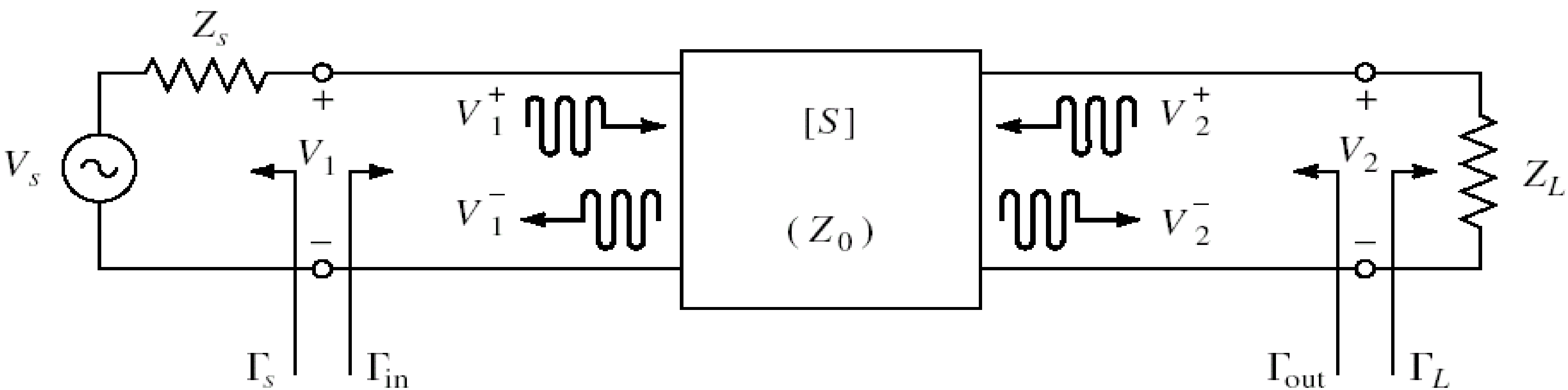


Figure 7.1: A two port network with general source and load impedance.

TWO-PORT POWER GAIN

Power Gain = $G = P_L / P_{in}$ is the ratio of power dissipated in the load Z_L to the power delivered to the input of the two-port network. This gain is independent of Z_s although some active circuits are strongly dependent on Z_S.

Available Gain = $G_A = P_{avn} / P_{avs}$ is the ratio of the power available from the two-port network to the power available from the source. This assumes conjugate matching in both the source and the load, and depends on Z_S but not Z_L.

Transducer Power Gain = $G_T = P_L / P_{avs}$ is the ratio of the power delivered to the load to the power available from the source. This depends on both Z_S and Z_L.

If the input and output are both conjugately matched to the two-port, then the gain is maximized and $G = G_A = G_T$

TWO-PORT POWER GAIN

From the definition of S parameters:

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ + S_{12}\Gamma_L V_2^- \qquad [3.1a]$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ + S_{22}\Gamma_L V_2^- \qquad [3.1b]$$

Eliminating V_2^- from [3.1a]:

$$\boxed{\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = \frac{Z_{in}-Z_0}{Z_{in}+Z_0}} \qquad [3.2]$$

$$\boxed{\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} = \frac{Z_{out}-Z_0}{Z_{out}+Z_0}} \qquad [3.3]$$

TWO-PORT POWER GAIN

By voltage division:

$$V_1 = V_S \frac{Z_{in}}{Z_S + Z_{in}} = V_1^+ + V_1^- = V_1^+\left(1 + \Gamma_{in}\right) \qquad [3.4]$$

Using:

$$\boxed{Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}}} \qquad [3.5]$$

Solving for V_1^+:

$$V_1^+ = \frac{V_S}{2} \frac{\left(1-\Gamma_S\right)}{\left(1-\Gamma_S\Gamma_{in}\right)} \qquad [3.6]$$

TWO-PORT POWER GAIN

The average power delivered to the network:

$$P_{in} = \frac{1}{2Z_0}\left|V_1^+\right|\left(1-\left|\Gamma_{in}\right|^2\right) = \frac{\left|V_S\right|^2}{8Z_0}\frac{\left|1-\Gamma_S\right|^2}{\left|1-\Gamma_S\Gamma_{in}\right|^2}\left(1-\left|\Gamma_{in}\right|^2\right) \qquad [3.7]$$

The power delivered to the load is:

$$P_L = \frac{\left|V_2^-\right|^2}{2Z_0}\left(1-\left|\Gamma_L\right|^2\right) \qquad [3.8]$$

$$P_L = \frac{\left|V_1^+\right|^2}{2Z_0}\frac{\left|S_{21}\right|^2\left(1-\left|\Gamma_L\right|^2\right)}{\left|1-S_{22}\Gamma_L\right|^2}$$

$$= \frac{\left|V_S\right|^2}{8Z_0}\frac{\left|S_{21}\right|^2\left(1-\left|\Gamma_L\right|^2\right)\left|1-\Gamma_S\right|^2}{\left|1-S_{22}\Gamma_L\right|^2\left|1-\Gamma_S\Gamma_{in}\right|^2} \qquad [3.9]$$

TWO-PORT POWER GAIN

The power gain can be expressed as:

$$\boxed{G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{\left(1-|\Gamma_{in}|^2\right)|1-S_{22}\Gamma_L|^2}} \quad [3.10]$$

The available power from the source:

$$P_{avs} = P_{in}\Big|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_s|^2}{8Z_0}\,\frac{|1-\Gamma_S|^2}{\left(1-|\Gamma_S|^2\right)} \quad [3.11]$$

The available power from the network:

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_s|^2}{8Z_0}\,\frac{|S_{21}|^2\left(1-|\Gamma_{out}|^2\right)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_{out}^*|^2\,|1-\Gamma_S\Gamma_{in}|^2}\Bigg|_{\Gamma_L=\Gamma_{out}^*} \quad [3.12]$$

TWO-PORT POWER GAIN

The power available from the network:

$$P_{avn} = \frac{|V_s|^2}{8Z_0} \frac{|S_{21}|^2 |1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2 \left(1-|\Gamma_{out}|^2\right)} \quad [3.13]$$

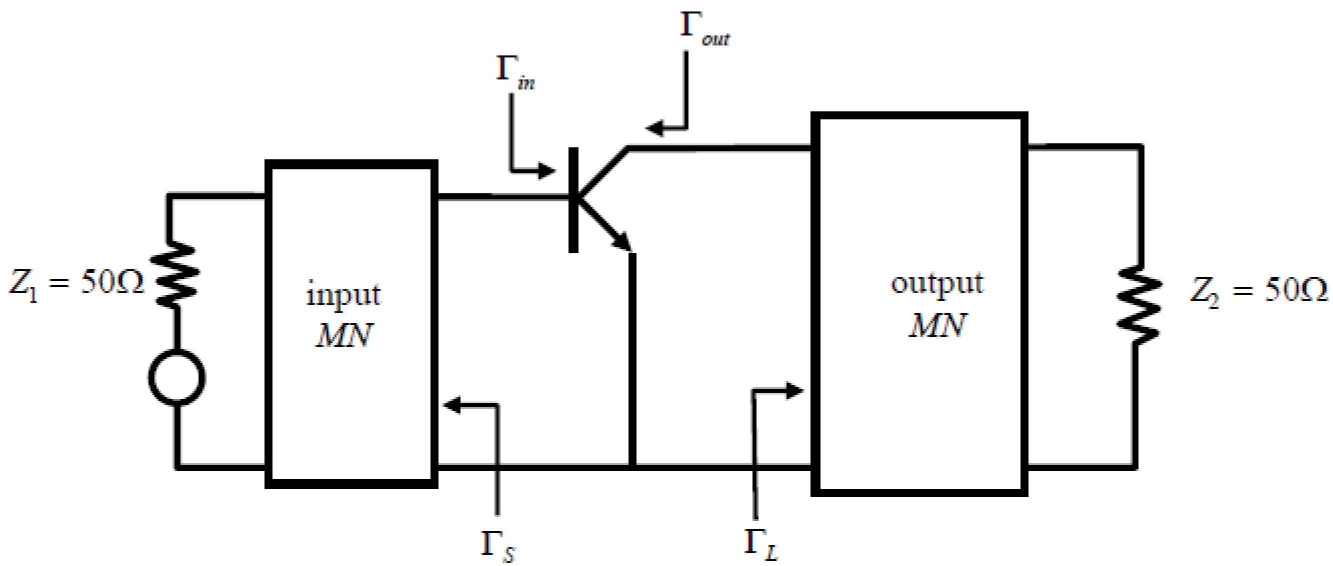
The available power gain:

$$\boxed{G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2 \left(1-|\Gamma_S|^2\right)}{\left(1-|\Gamma_{out}|^2\right)|1-S_{11}\Gamma_S|^2}} \quad [3.14]$$

The transducer power gain:

$$\boxed{G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2 \left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-S_{22}\Gamma_L|^2 |1-\Gamma_S\Gamma_{in}|^2}} \quad [3.15]$$

AMPLIFIER BLOCK DIAGRAM



Unilateral case

If the transistor is unilateral,

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2},$$
$$G_0 = |S_{21}|^2,$$
$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}.$$

Evaluate the appropriate gain equation: $G_T =$ transducer power gain $= \dfrac{P_L}{P_{AVS}}$

$$G_T = \underbrace{\frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}}_{\text{gain term associated with input match}} \quad \underbrace{|S_{21}|^2}_{\uparrow\ G_T \text{ of device if } \Gamma_S = \Gamma_L = Z_0} \quad \underbrace{\frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{out}\Gamma_L|^2}}_{\text{gain term associated with output match}}$$

where $\Gamma_{out} = S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$

So, if you are given the S params and Γ_S, Γ_L then you can calculate the gain.

Note however that Γ_{out} depends on Γ_S unless $S_{12} = 0$!

Further discussion of two-ports power gains

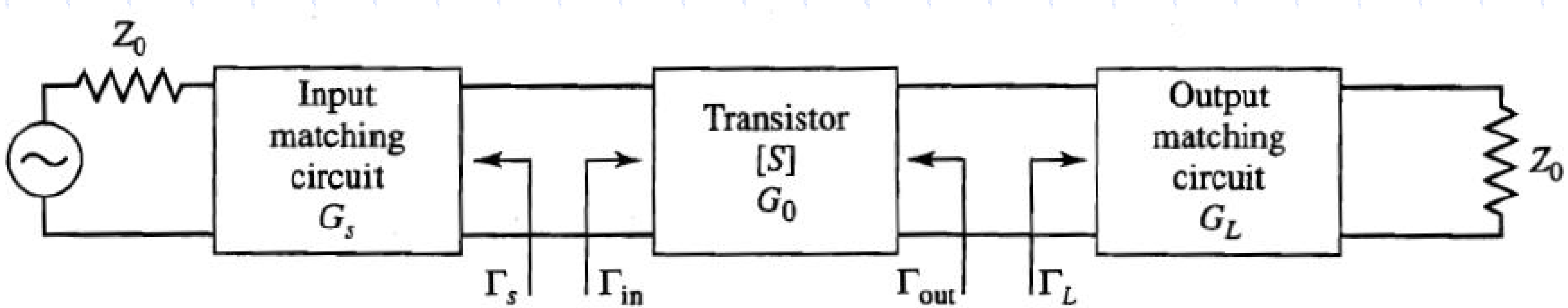
we can define separate effective gain factors for the input (source) matching network, the transistor itself, and the output (load) matching network as follows:

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2},$$

$$G_0 = |S_{21}|^2,$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}.$$

$$G_T = G_S G_0 G_L$$



The general transistor amplifier circuit.

Special case 1

$$\Gamma_L = \Gamma_S = 0$$

$$G_T = |S_{21}|^2.$$

Special case 2: S_{12}=0, unilateral transducer power gain

$$G_{\mathrm{TU}} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - S_{11}\Gamma_S|^2|1 - S_{22}\Gamma_L|^2}$$

EXAMPLE COMPARISON OF POWER GAIN DEFINITIONS

A microwave transistor has the following S parameters at 10 GHz, with a 50 Ω reference impedance:

$$S_{11} = 0.45\angle 150^\circ$$
$$S_{12} = 0.01\angle -10^\circ$$
$$S_{21} = 2.05\angle 10^\circ$$
$$S_{22} = 0.40\angle -150^\circ$$

The source impedance is $Z_S = 20\ \Omega$ and the load impedance is $Z_L = 30\ \Omega$. Compute the power gain, the available gain, and the transducer power gain.

Solution

From (6.4a,b) the reflection coefficients at the source and load are

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{20 - 50}{20 + 50} = -0.429,$$
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - 50}{30 + 50} = -0.250.$$

the reflection coefficients seen looking at the input and output of the terminated network are

$$\begin{aligned}
\Gamma_{\text{in}} &= S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \\
&= 0.45\angle 150^\circ + \frac{(0.01\angle -10^\circ)(2.05\angle 10^\circ)(-0.250)}{1-(0.40\angle -150^\circ)(-0.250)} = 0.455\angle 150^\circ, \\
\Gamma_{\text{out}} &= S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \\
&= 0.40\angle -150^\circ + \frac{(0.01\angle -10^\circ)(2.05\angle 10^\circ)(-0.429)}{1-(0.45\angle 150^\circ)(-0.429)} = 0.408\angle -151^\circ.
\end{aligned}$$

the power gain is

$$G = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - |\Gamma_{\text{in}}|^2)|1 - S_{22}\Gamma_L|^2}$$

$$= \frac{(2.05)^2[1 - (0.250)^2]}{|1 - (0.40\angle{-150°})(-0.250)|^2[1 - (0.455)^2]} = 5.94.$$

the available power gain is

$$G_A = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{|1 - S_{11}\Gamma_S|^2(1 - |\Gamma_{\text{out}}|^2)}$$

$$= \frac{(2.05)^2[1 - (0.429)^2]}{|1 - (0.45\angle 150°)(-0.429)|^2[1 - (0.408)^2]} = 5.85.$$

the transducer power gain is

$$G_T = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_S\Gamma_{\text{in}}|^2|1 - S_{22}\Gamma_L|^2}$$

$$= \frac{(2.05)^2[1 - (0.429)^2][1 - (0.250)^2]}{|1 - (0.40\angle{-150°})(-0.250)|^2|1 - (-0.429)(0.455\angle 150°)|^2} = 5.49$$

Örnek:

2GHz amplifier (Z_o=50Ω) with S_{11}=0.97$\angle$-43°, S_{12}=0,
S_{21}=3.39$\angle$140°, S_{22}=0.63$\angle$-32°, Γ_s=0.97$\angle$43°, Γ_L=0.63$\angle$32°,
$\rightarrow$ G_T, G_P, G_A

$$S_{12} = 0 \rightarrow \Gamma_{in} = S_{11}, \Gamma_{out} = S_{22}$$

$$\Gamma_s = S_{11}^*, \Gamma_L = S_{22}^*$$

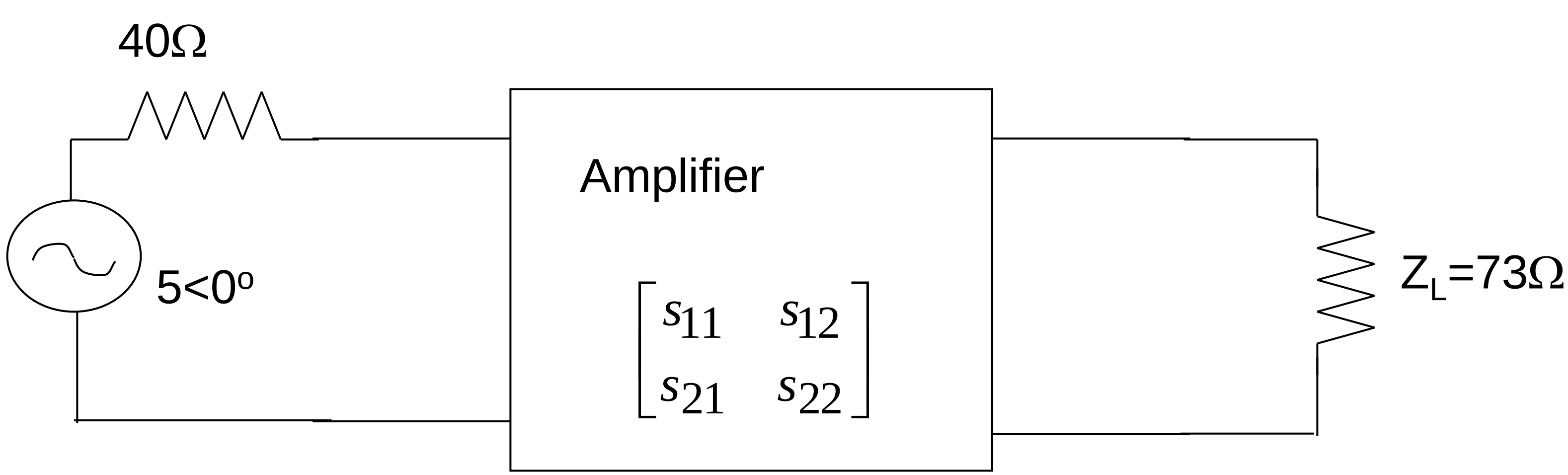
$$G_P \equiv \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2}, G_A \equiv \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{(1-|\Gamma_{out}|^2)|1-S_{11}\Gamma_s|^2}$$

$$G_T \equiv \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_s\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2}$$

$$G_T = G_A = G_P = G_{TU\,\max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} = 322.42 = 25dB$$

Example: Gain Expressions

- An RF amplifier has the following S-parameters at f_o: $s_{11}=0.3\angle{-70^o}$, $s_{21}=3.5\angle{85^o}$, $s_{12}=0.2\angle{-10^o}$, $s_{22}=0.4\angle{-45^o}$. The system is shown below. Assuming reference impedance (used for measuring the S-parameters) $Z_o=50\Omega$, find:
- (a) G_T, G_A, G_P.
- (b) P_L, P_A, P_{inc}.



Example Cont...

- Step 1 - Find Γ_s and Γ_L .
- Step 2 - Find Γ_1 and Γ_2 .
- Step 3 - Find G, G_A, G_T.
- Step 4 - Find P_L, P_A.
- D=s11s22-s12s21
- Gp=GT

$$P_A = \frac{|V_s|^2}{8\cdot \mathrm{Re}[Z_s]} = 0.078W$$

Try to derive

These 2 relations

$$P_{in} = P_A\left(1-\left|\frac{Z_1-Z_s}{Z_1+Z_s}\cdot Z_o\right|^2\right) = 0.0714W$$

$$P_L = G_P P_{in} = 0.9814W$$

Note that this is an analysis problem.

$$\Gamma_s = \frac{Z_s-Z_o}{Z_s+Z_o} = -0.111 \qquad \Gamma_L = \frac{Z_L-Z_o}{Z_L+Z_o} = 0.187$$

$$\Gamma_1 = \frac{s_{11}-D\Gamma_L}{1-s_{22}\Gamma_L} = 0.146 - j0.151$$

$$\Gamma_2 = \frac{s_{22}-D\Gamma_s}{1-s_{11}\Gamma_s} = 0.265 - j0.358$$

$$G = \frac{|s_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-s_{22}\Gamma_L|^2\left(1-|\Gamma_1|^2\right)} = 13.742$$

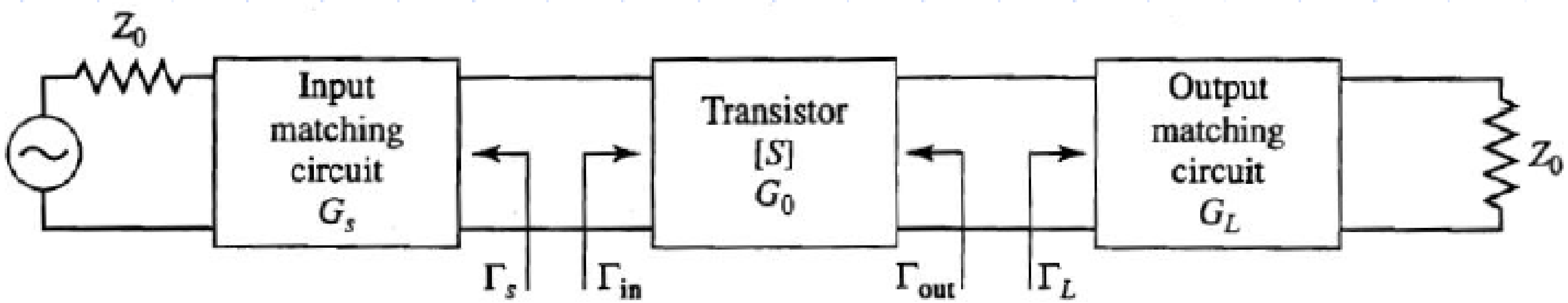
$$G_A = \frac{\left(1-|\Gamma_s|^2\right)|s_{21}|^2}{|1-s_{11}\Gamma_s|^2\left(1-|\Gamma_2|^2\right)} = 14.739$$

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|s_{21}|^2\left(1-|\Gamma_s|^2\right)}{|1-s_{22}\Gamma_L|^2|1-\Gamma_1\Gamma_s|^2} = 12.562$$

Stability

Oscillation is possible if either the input or output port impedance has a negative real part; this would then imply that $|\Gamma_{in}| > 1$ or $|\Gamma_{out}| > 1$.

Because Γ_{in} and Γ_{out} depend on the source and load matching networks, the stability of the amplifier depends on Γ_S and Γ_L



The general transistor amplifier circuit.

- *Unconditional stability*: The network is unconditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for all passive source and load impedances (i.e., $|\Gamma_S| < 1$ and $|\Gamma_L| < 1$).
- *Conditional stability*: The network is conditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ only for a certain range of passive source and load impedances. This case is also referred to as *potentially unstable*.

Load stability circle

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \qquad \text{(center)},$$

$$R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \qquad \text{(radius)}.$$

Source stability circle

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \qquad \text{(center)},$$

$$R_S = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \qquad \text{(radius)}.$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}.$$

Unconditional stable:

If the device is unconditionally stable, the stability circles must be completely outside (or totally enclose) the Smith chart.

$$||C_L| - R_L| > 1, \qquad \text{for } |S_{11}| < 1,$$
$$||C_S| - R_S| > 1, \qquad \text{for } |S_{22}| < 1.$$

If $|S_{11}| > 1$ or $|S_{22}| > 1$, the amplifier cannot be unconditionally stable because we can always have a source or load impedance of Z_0 leading to $\Gamma_S = 0$ or $\Gamma_L = 0$, thus causing $|\Gamma_{in}| > 1$ or $|\Gamma_{out}| > 1$. If the device is only conditionally stable, operating points for Γ_S and Γ_L must be chosen in stable regions, and it is good practice to check the stability at several frequencies near the design frequency. If it is possible to accept a design with less than maximum gain, a transistor can usually be made to be unconditionally stable by using resistive loading .

Tests for unconditional stability

K-Δ test

These two conditions are necessary and sufficient for unconditional stability, and are easily evaluated.

$$\longrightarrow K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1,$$

$$\longrightarrow |\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1,$$

→ recall from the previous paragraph that we must have $\boxed{|S_{11}| < 1 \text{ and } |S_{22}| < 1}$ if the device is to be unconditionally stable.

While the K-Δ test is a mathematically rigorous condition for unconditional stability, it cannot be used to compare the relative stability of two or more devices since it involves constraints on two separate parameters. Recently, however, a new criterion has been proposed that combines the S parameters in a test involving only a single parameter, μ, defined as

A quantitative test

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|} > 1.$$

Thus, if $\mu > 1$, the device is unconditionally stable. In addition, larger values of μ imply greater stability.

$$\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = \frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S},$$

Unconditional stability implie that $|\Gamma_{\text{out}}| < 1$ for any passive source termination, Γ_S.

Example 2

The S parameters for the HP HFET-102 GaAs FET at 2 GHz with a bias voltage of Vgs = 0 are given as follow (Z_0 = 50 Ohm):

$$S_{11} = 0.894 < -60.6$$
$$S_{21} = 3.122 < 123.6$$
$$S_{12} = 0.020 < 62.4$$
$$S_{22} = 0.781 < -27.6$$

Determine the stability of this transistor using the K-Δ test and the μ test, and plot the stability circles on the Smith Chart

Example 2

Remember, criteria for unconditional stability is:

For the *K*-*Δ* test:

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1$$

For the μ test:

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|} > 1$$

Example 2

Calculation results:

For the K-Δ test:

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| = 0.696 < 1$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 0.607 < 1$$

For the μ test:

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|} = 0.86 < 1$$

Which indicates potential instability

Example 2

Calculation for the input and output stability circles:

Output stability circle center and radius:

$$C_L = \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} = 1.361 < 47$$

$$R_L = \left| \frac{S_{12} S_{21}}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} \right| = 0.50$$

Input stability circle and radius

$$C_S = \frac{\left(S_{11} - \Delta S_{22}^*\right)^*}{\left|S_{11}\right|^2 - \left|\Delta\right|^2} = 1.132 < 68$$

$$R_S = \left| \frac{S_{12} S_{21}}{\left|S_{11}\right|^2 - \left|\Delta\right|^2} \right| = 0.199$$

STABILITY

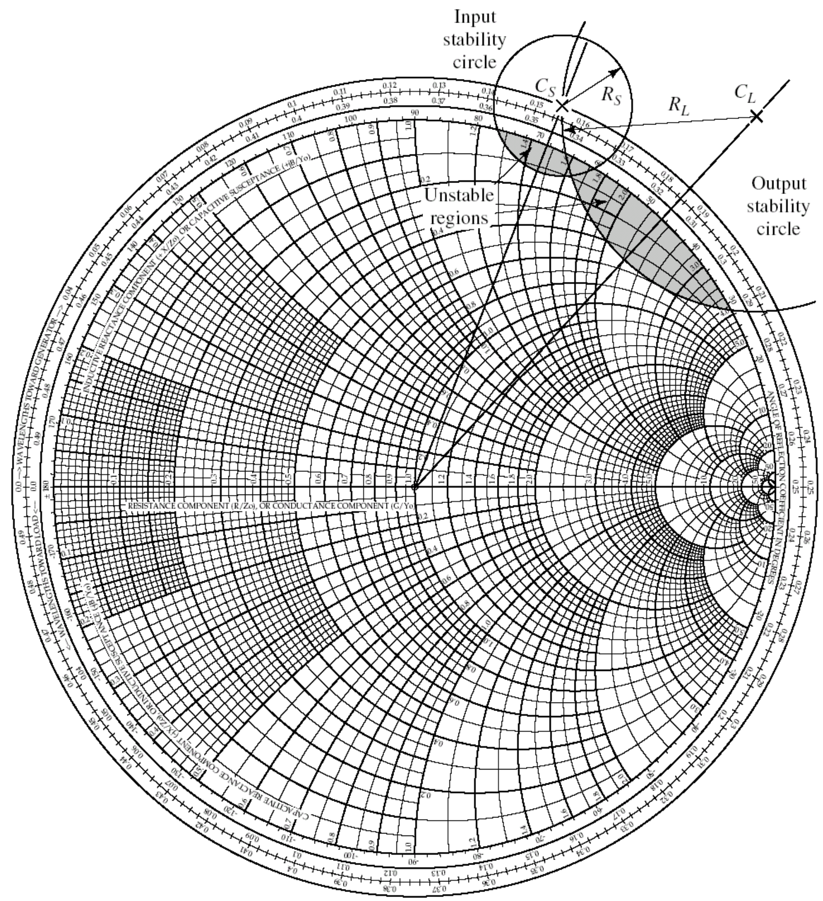


Figure 7.4: Example of stability circles

A unilateral device condition for unconditional stability

- For a unilateral device condition for unconditional stability in terms of S parameters is $|S_{11}|<1$, $|S_{22}|<1$.

  Explanation: For a unilateral device, the condition for unconditional stability is $|S_{11}|<1$, $|S_{22}|<1$. S_{11} parameter signifies the amount of power reflected back to port 1, which is the input port of the transistor. If this S parameter is greater is than 1, more amount of power is reflected back implying the amplifier is unstable.
- If $|S_{11}|>1$ or $|S_{22}|>1$, the amplifier cannot be unconditionally stable.

  If $|S_{11}|>1$ or $|S_{22}|>1$, the amplifier cannot be unconditionally stable because we can have a source or load impedance of Z_o leading to $\Gamma s=0$ or $\Gamma L=0$, thus causing output and input reflection coefficients greater than 1.

Condition for unconditional stability

- The condition for unconditional stability of a transistor is $|\Delta| < 1$ and $K > 1$. Here, $|\Delta|$ and K are defined in terms of the s parameters of the transistor by defining the S matrix. To determine the unconditional stability of a transistor in K-Δ method, the S matrix of the transistor must be known.
- If the S parameters of a transistor given are
  S_{11}=-0.811-j0.311
  S_{12}= 0.0306+j0.0048
  S_{21}=2.06+j3.717
  S_{22}=-0.230-j0.4517
  Then Δ for the given transistor is 0.336.

  Explanation: Given the S parameters of a transistor, the Δ value of the transistor is given by $|S_{11}S_{22}-S_{12}S_{21}|$. Substituting the given values in the above equation, the Δ of the transistor is 0.336.

Unilateral Gain

Unilateral Gain

This time the mismatch factors, G_S and G_L, are independent of each other, meaning that the input and output ports of the device can be matched independently. G_S and G_L result from (13) and (14) when we set $\Gamma_{in} = S_{11}$ and $\Gamma_{out} = S_{22}$. G_o simply represents the transducer gain of the active device when terminated in the system characteristic impedance, Z_o.
It is a simple matter to show that, for a unilateral device, maximum gain is obtained when we set:

$$\Gamma_S = S_{11}^*$$

and

$$\Gamma_L = S_{22}^*$$

(17) then gives the value of the maximum unilateral gain as:

$$G_{TU_{max}} = \frac{1}{|1 - |S_{11}|^2|} |S_{21}|^2 \frac{1}{|1 - |S_{22}|^2|}$$

A further useful consequence of assuming unilaterality is that Rollett's stability factor tends to infinity and the stability criteria (**??**) simply reduce to:

$$|S_{11}| < 1$$
$$|S_{22}| < 1$$

Amplifier Design using S-parameters

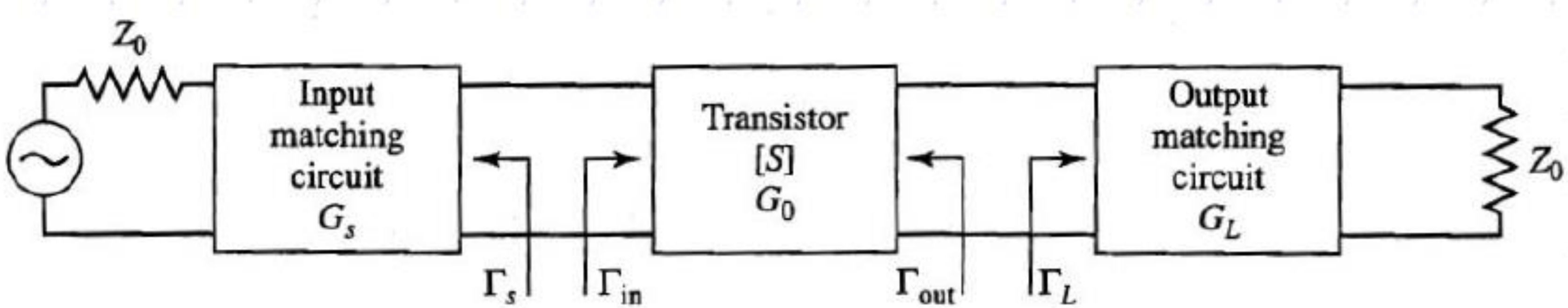
Amplifier Design using S-parameters

- Stability check
- Matching network design according to gain, power or noise maximization
- Conjugate match for maximum gain but usually lower bandwidth.
- Trade off between gain and bandwidth

$$\Gamma_{in} = \Gamma_S^*,$$

$$\Gamma_{out} = \Gamma_L^*.$$

$$G_{T_{max}} = \frac{1}{1-|\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}.$$



The general transistor amplifier circuit.

In the general case with a bilateral transistor ($|S_{12}| \neq 0$), Γ_{in} is affected by Γ_{out}, and vice versa, so that the input and output sections must be matched simultaneously.
the necessary equations:

$$\Gamma_S^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L},$$

$$\Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}.$$

We can solve for Γ_S by first rewriting these equations as follows:

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* S_{21}^*}{1/\Gamma_L^* - S_{22}^*},$$

$$\Gamma_L^* = \frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S},$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$, as before.

$$\boxed{\begin{aligned} &\Gamma_S(1 - |S_{22}|^2) + \Gamma_S^2(\Delta S_{22}^* - S_{11}) \\ &\quad = \Gamma_S(\Delta S_{11}^* S_{22}^* - |S_{11}|^2 - \Delta S_{12}^* S_{21}^*) + S_{11}^*(1 - |S_{22}|^2) + S_{12}^* S_{21}^* S_{22} \end{aligned}}$$

$$(S_{11} - \Delta S_{22}^*)\Gamma_S^2 + (|\Delta|^2 - |S_{11}|^2 + |S_{22}|^2 - 1)\Gamma_S + (S_{11}^* - \Delta^* S_{22}) = 0.$$

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}.$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}.$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta^2|,$$
$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta^2|,$$
$$C_1 = S_{11} - \Delta S_{22}^*,$$
$$C_2 = S_{22} - \Delta S_{11}^*.$$

Solutions to

Γ_S and Γ_L are only possible if the quantity within the square root is positive, and it can be shown that this is equivalent to requiring K > 1.

Thus unconditionally stable devices can always be conjugately matched for maximum gain, and potentially unstable devices can be conjugately matched if K > 1 and $|\Delta| < 1$.

for the unilateral case. When $S_{12} = 0$, $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$, and then the maximum transducer gain of reduces to

$$G_{TU_{\max}} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2}.$$

Maximum stable gain

If the transistor is unconditionally stable, so that K > 1, the maximum transducer power gain of (6.40) can be simply rewritten as follows:

$$G_{T_{\max}} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right).$$

The maximum gain does not provide a meaningful result if the device is only conditionally stable, since simultaneous conjugate matching of the source and load are not possible if K < 1

In this case a useful figure of merit is the maximum stable gain, defined as the maximum transducer power gain of (6.46) with K = 1.

For K = 1:

$$G_{\mathrm{msg}} = \frac{|S_{21}|}{|S_{12}|}.$$

The maximum stable gain is easy to compute and offers a convenient way to compare the gain of various devices under stable operating conditions.

CONJUGATELY MATCHED AMPLIFIER DESIGN

Design an amplifier for maximum gain at 4.0 GHz using single-stub matching sections. Calculate and plot the input return loss and the gain from 3 to 5 GHz. Use a GaAs FET with the following S parameters ($Z_0 = 50\ \Omega$):

f (GHz)	S_{11}	S_{21}	S_{12}	S_{22}
3.0	$0.80\angle -89°$	$2.86\angle 99°$	$0.03\angle 56°$	$0.76\angle -41°$
4.0	$0.72\angle -116°$	$2.60\angle 76°$	$0.03\angle 57°$	$0.73\angle -54°$
5.0	$0.66\angle -142°$	$2.39\angle 54°$	$0.03\angle 62°$	$0.72\angle -68°$

Solution

We first check for unconditional stability of the transistor by calculating Δ and K at 4.0 GHz:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.488\angle -162°,$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.195.$$

Since $|\Delta| < 1$ and $K > 1$, the transistor is unconditionally stable at 4.0 GHz. There is therefore no need to plot the stability circles.

For maximum gain, we should design the matching sections for a conjugate match to the transistor. Thus, $\Gamma_S = \Gamma_{\text{in}}^*$ and $\Gamma_L = \Gamma_{\text{out}}^*$, and Γ_S and Γ_L can be determined :

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} = 0.872\angle 123^\circ$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} = 0.876\angle 61^\circ.$$

Then the effective gain factors can be calculated as

$$G_S = \frac{1}{1 - |\Gamma_S|^2} = 4.17 = 6.20 \text{ dB},$$

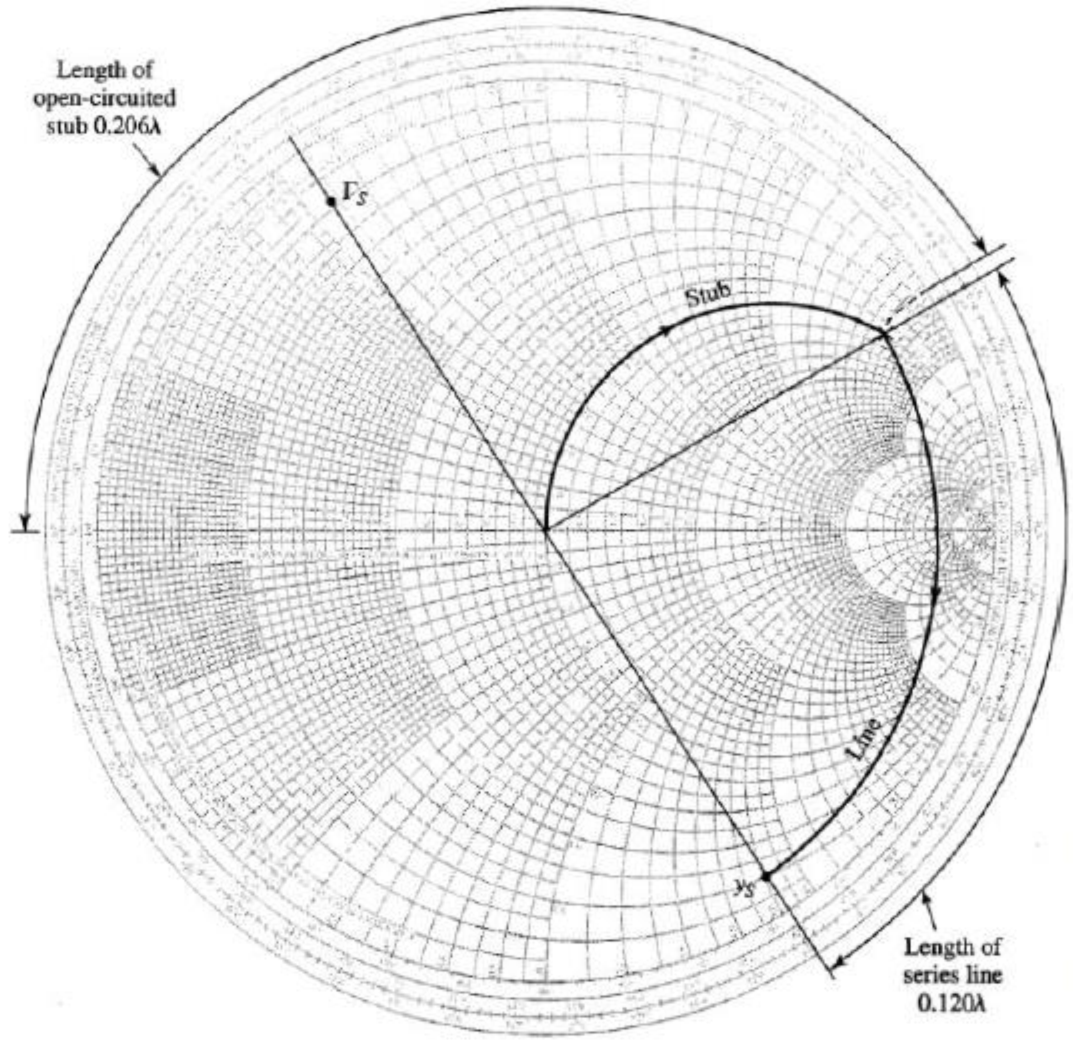
$$G_0 = |S_{21}|^2 = 6.76 = 8.30 \text{ dB},$$

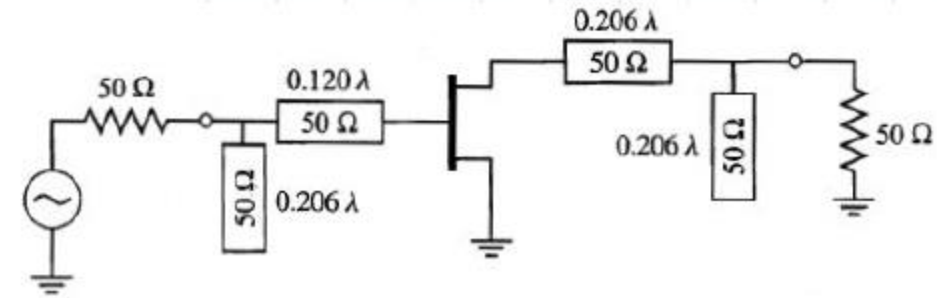
$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = 1.67 = 2.22 \text{ dB}.$$

So the overall transducer gain will be

$$G_{T_{\max}} = 6.20 + 8.30 + 2.22 = 16.7 \text{ dB}.$$

The matching networks can easily be determined with a Smith chart. For the input matching section, we first plot Γ_S,


Length of
open-circuited
stub 0.206λ
Γ_S
Stub
Line
y_S
Length of
series line
0.120λ


50 Ω
0.120 λ
50 Ω
50 Ω
0.206 λ
0.206 λ
50 Ω
0.206 λ
50 Ω
50 Ω

Example

Design an amplifier for a maximum gain at 4.0 GHz. Calculate the overall transducer gain, *G*, and the maximum overall transducer gain G_{TMAX}. The S parameters for the GaAs FET at 4 GHz given as follow (Z_0 = 50 Ohm):

$$S_{11} = 0.72 < -116$$
$$S_{21} = 2.60 < 76$$
$$S_{12} = 0.03 < 57$$
$$S_{22} = 0.73 < -68$$

Example (Cont)

Determine the stability of this transistor using the *K*-*Δ* test

$$S_{11}S_{22} - S_{12}S_{21} = 0.488 < -162$$

$$\left|\Delta\right| = 0.488$$

$$K = \frac{1 - \left|S_{11}\right|^2 - \left|S_{22}\right|^2 + \left|\Delta\right|^2}{2\left|S_{12}S_{21}\right|} = 1.195$$

Since $|\Delta| < 1$ and $K > 1$, the transistor is unconditionally stable at 4.0 GHz.

Example (cont)

For the maximum gain, we should design the matching sections for a conjugate match to the transistor. Thus, $\Gamma_S = \Gamma_{in}^*$ and $\Gamma_L = \Gamma_{out}^*$, Γ_S and Γ_L can be determined from;

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_2^2 - 4|C_1|^2}}{2C_1} = 0.872 < 123$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_1^2 - 4|C_2|^2}}{2C_2} = 0.876 < 61$$

Example

The effective gain factors can calculated as:

$$G_S = \frac{1}{1-|S_{11}|^2} = 4.17 = 6.20dB$$

$$G_0 = |S_{21}|^2 = 6.76 = 8.30dB$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = 1.67 = 2.22dB$$

So the overall maximum transducer gain will be;

$$G_{T\max} = 6.20 + 8.30 + 2.22 = 16.7dB$$

Example 4

An FET is biased for minimum noise figure, and has the following S parameters at 4 GHz:

$$S_{11} = 0.60 < -60$$
$$S_{21} = 1.90 < 81$$
$$S_{12} = 0.05 < 26$$
$$S_{22} = 0.50 < -60$$

For design purposes, assume the device is unilateral and calculate the max error in G_T resulting from this assumption.

Unilateral form

- In many practical cases $|S_{12}|$ is small enough to be ignored, the device then can be assumed to be unilateral, which greatly simplifies design procedure
- Error in the transducer gain caused by approximating $|S_{12}|$ as zero is given by the ratio G_T/G_{TU}, and be bounded by:

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$

Where U is defined as the unilateral figure of merit

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

Example 4 (cont)

To compute the unilateral figure of merit;

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = 0.059$$

Then the ratio of G_T/G_{TU} is bounded as;

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$

$$0.891 < \frac{G_T}{G_{TU}} < 1.130$$

Example 4 (cont)

In dB, this is;

$$-0.50 < G_T - G_{TU} < 0.53dB$$

Where G_T and G_{TU} are now in dB. Thus we should expect less than about ± 0.5 dB error in gain.

CONSTANT GAIN CIRCLES

Constant gain circle and design for specified gain

- In many cases it is preferable to design for less than the maximum obtainable gain, to improve bandwidth, to obtain a specific value of amplifier gain, or to minimize the effect of device variations.
- This can be done by designing the input and output matching sections to have less than maximum gains; in other words, impedance mismatches are purposely introduced to reduce the overall gain.
- The design procedure is facilitated by plotting constant gain circles on a Smith chart, to represent loci of Γ_S and Γ_L that give fixed values of gain for the input and output sections (Gs and GL).

Constant gain circle and design for specified gain

Only unilateral case is shown for simplicity

The error in the

transducer gain caused by approximating $|S_{12}|$ as zero is given by the ratio G_T/G_{TU}. It can be shown that this ratio is bounded by

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}, \tag{6.48}$$

where U is defined as the *unilateral figure of merit*,

$$U = \frac{|S_{11}||S_{12}||S_{21}||S_{22}|}{(1-|S_{11}|^2)(1-S_{22}|^2)}. \tag{6.49}$$

Usually an error of a few tenths of a dB or less will justify the unilateral assumption.

The expressions for G_S and G_L for the unilateral case are given by (6.20a) and (6.20c):

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2},$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}.$$

These gains are maximized when $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$, resulting in the maximum values given by

$$G_{S_{\max}} = \frac{1}{1 - |S_{11}|^2}, \tag{6.50a}$$

$$G_{L_{\max}} = \frac{1}{1 - |S_{22}|^2}. \tag{6.50b}$$

Now define the normalized gain factors g_S and g_L as

$$g_S = \frac{G_S}{G_{S_{\max}}} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}(1 - |S_{11}|^2), \tag{6.51a}$$

$$g_L = \frac{G_L}{G_{L_{\max}}} = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}(1 - |S_{22}|^2). \tag{6.51b}$$

Then we have that $0 \leq g_S \leq 1$, and $0 \leq g_L \leq 1$.

For fixed values of g_S and g_L, (6.51) represents circles in the Γ_S or Γ_L plane. To show this, consider (6.51a), which can be expanded to give

$$g_S|1 - S_{11}\Gamma_S|^2 = (1 - |\Gamma_S|^2)(1 - |S_{11}|^2),$$

$$(g_S|S_{11}|^2 + 1 - |S_{11}|^2)|\Gamma_S|^2 - g_S\left(S_{11}\Gamma_S + S_{11}^*\Gamma_S^*\right) = 1 - |S_{11}|^2 - g_S,$$

$$\Gamma_S\Gamma_S^* - \frac{g_S(S_{11}\Gamma_S + S_{11}^*\Gamma_S^*)}{1 - (1 - g_S)|S_{11}|^2} = \frac{1 - |S_{11}|^2 - g_S}{1 - (1 - g_S)|S_{11}|^2}. \tag{6.52}$$

$$\left|\Gamma_S - \frac{g_S S_{11}^*}{1 - (1 - g_S)|S_{11}|^2}\right| = \frac{\sqrt{1 - g_S}(1 - |S_{11}|^2)}{1 - (1 - g_S)|S_{11}|^2}, \tag{6.53}$$

The equation of a circle with its center and radius given by

$$C_S = \frac{g_S S_{11}^*}{1 - (1 - g_S)|S_{11}|^2}, \tag{6.54a}$$

$$R_S = \frac{\sqrt{1 - g_S}(1 - |S_{11}|^2)}{1 - (1 - g_S)|S_{11}|^2}. \tag{6.54b}$$

output section

$$C_L = \frac{g_L S_{22}^*}{1 - (1 - g_L)|S_{22}|^2}, \tag{6.55a}$$

$$R_L = \frac{\sqrt{1 - g_L}(1 - |S_{22}|^2)}{1 - (1 - g_L)|S_{22}|^2}. \tag{6.55b}$$

Summary for the constant gain circles

The centers of each family of circles lie along straight lines given by the angle of S_{11}^* or S_{22}^*.

when g_S (or g_L) = 1 (maximum gain), the radius R_S (or R_L) = 0, and the center reduces to S_{11}^* (or S_{22}^*)

it can be shown that the 0 dB gain circles
($G_S = 1$ or $G_L = 1$) will always pass through the center of the Smith chart.

The choices
for Γ_S and Γ_L are not unique, but it makes sense to choose points close to the center of the Smith chart to minimize the mismatch, and thus maximize the bandwidth. Alternatively, as we will see in the next section, the input network mismatch can be chosen to provide a low-noise design.

AMPLIFIER DESIGN FOR SPECIFIED GAIN

Design an amplifier to have a gain of 11 dB at 4.0 GHz. Plot the constant gain circles for $G_S = 2$ dB and 3 dB, and $G_L = 0$ dB and 1 dB. Calculate and plot the input return loss and overall amplifier gain from 3 to 5 GHz. Use an FET with the following S parameters ($Z_0 = 50\ \Omega$):

f (GHz)	S_{11}	S_{21}	S_{12}	S_{22}
3	$0.80\angle{-90^\circ}$	$2.8\angle 100^\circ$	0	$0.66\angle{-50^\circ}$
4	$0.75\angle{-120^\circ}$	$2.5\angle 80^\circ$	0	$0.60\angle{-70^\circ}$
5	$0.71\angle{-140^\circ}$	$2.3\angle 60^\circ$	0	$0.58\angle{-85^\circ}$

Solution

Since $S_{12} = 0$ and $|S_{11}| < 1$ and $|S_{22}| < 1$, the transistor is unilateral and unconditionally stable. From (6.50) we calculate the maximum matching section gains as

$$G_{S_{\max}} = \frac{1}{1 - |S_{11}|^2} = 2.29 = 3.6 \text{ dB},$$

$$G_{L_{\max}} = \frac{1}{1 - |S_{22}|^2} = 1.56 = 1.9 \text{ dB}.$$

The gain of the mismatched transistor is

$$G_0 = |S_{21}|^2 = 6.25 = 8.0 \text{ dB},$$

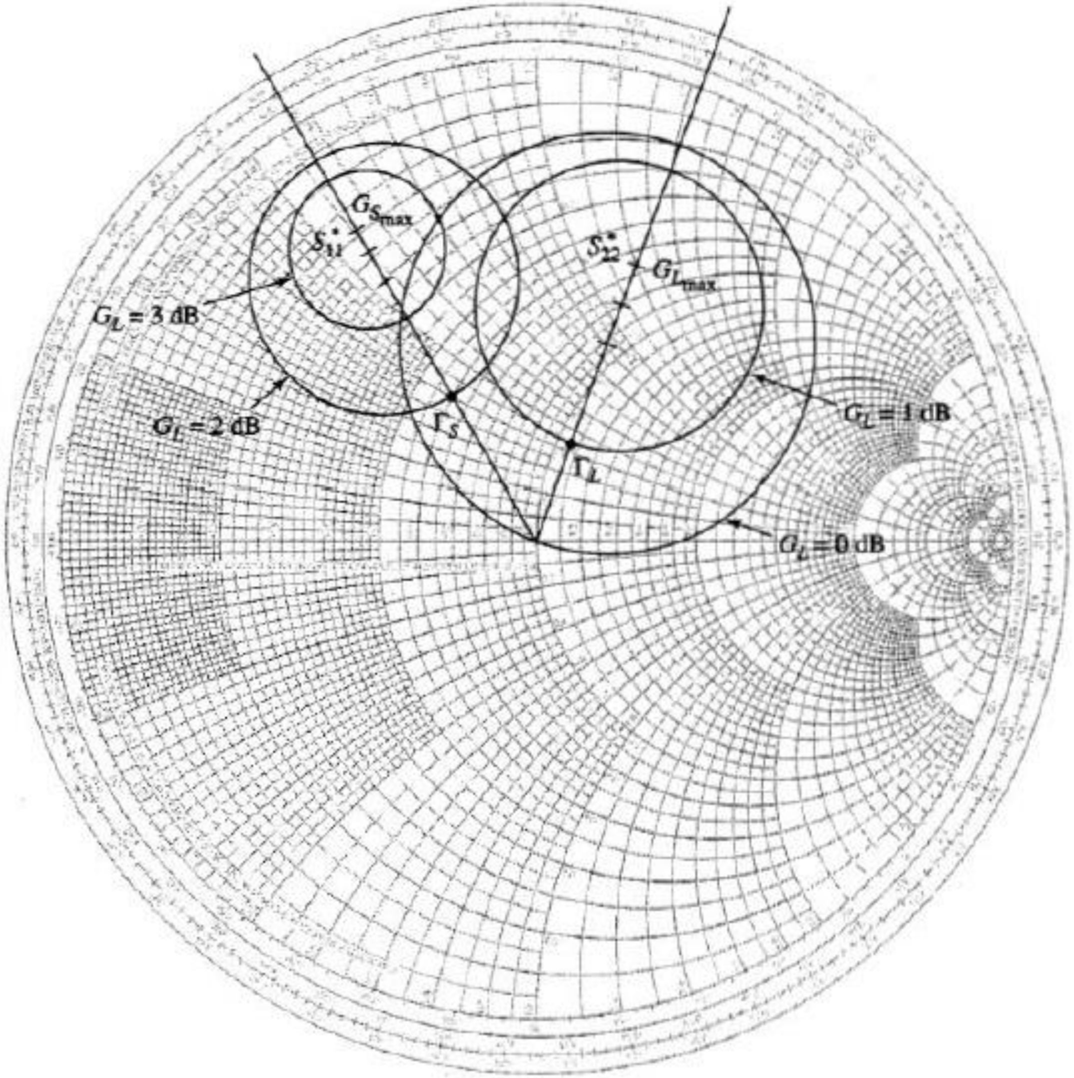
so the maximum unilateral transducer gain is

$$G_{TU_{\max}} = 3.6 + 1.9 + 8.0 = 13.5 \text{ dB}.$$

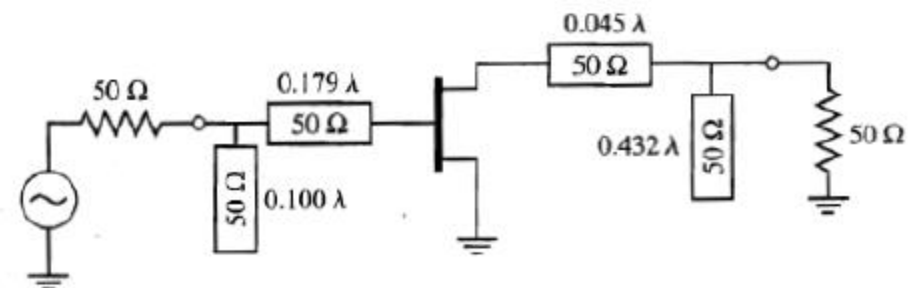
Thus we have 2.5 dB more gain than is required by the specifications. We use (6.51), (6.54), and (6.55) to calculate the following data for the constant gain circles:

$$\begin{array}{llll}
G_S = 3 \text{ dB} & g_S = 0.875 & C_S = 0.706\angle 120^\circ & R_S = 0.166 \\
G_S = 2 \text{ dB} & g_S = 0.691 & C_S = 0.627\angle 120^\circ & R_S = 0.294 \\
G_L = 1 \text{ dB} & g_L = 0.806 & C_L = 0.520\angle 70^\circ & R_L = 0.303 \\
G_L = 0 \text{ dB} & g_L = 0.640 & C_L = 0.440\angle 70^\circ & R_L = 0.440
\end{array}$$

The constant gain circles are shown in Figure 6.13a. We can choose $G_S = 2$ dB and $G_L = 1$ dB, for an overall amplifier gain of 11 dB. Then we select Γ_S and Γ_L along these circles as shown, to minimize the distance from the center of the chart (this places Γ_S and Γ_L along the radial lines at 120° and 70°, respectively). Thus,



$G_{S_{max}}$
S_{11}^*
S_{22}^*
$G_{L_{max}}$
$G_L = 3$ dB
$G_L = 2$ dB
$G_L = 1$ dB
$G_L = 0$ dB
Γ_S
Γ_L



50 Ω
0.179 λ
50 Ω
50 Ω
0.100 λ
0.045 λ
50 Ω
0.432 λ
50 Ω
50 Ω

Example 5

Design an amplifier to have a gain of 11 dB at 4 GHz. Plot constant gain circles for G_S = 2 dB and 3 dB; and G_L = 0 dB and 1 dB. The FET has the following S parameters (Z_0 = 50 Ω):

$$S_{11} = 0.75 < -120$$
$$S_{21} = 2.50 < 80$$
$$S_{12} = 0.00 < 0$$
$$S_{22} = 0.60 < -85$$

Example 5 (cont)

Since S_{12} = 0 and $|S_{11}|$ < 1 and $|S_{22}|$ < 1, the transistor is unilateral and unconditionally stable. We calculate the max matching section gains as;

$$G_{S\max} = \frac{1}{1-|S_{11}|^2} = 2.29 = 3.6dB$$

$$G_{L\max} = \frac{1}{1-|S_{22}|^2} = 1.56 = 1.9dB$$

The gain of the mismatched transistor is;

$$G_0 = |S_{21}|^2 = 6.25 = 8.0dB$$

Example 5 (cont)

So the max unilateral transducer gain is

$$G_{TU\max} = 3.6 + 1.9 + 8.0 = 13.5dB$$

Thus we have 2.5 dB more available gain than required by specs, since the design only requires 11 dB gain. However, the question also asked us to analyze the effect of having:

Condition 1: G_S = 3 dB and G_L = 0 dB
Condition 2: G_S = 2 dB and G_L = 1 dB

(Note that these conditions must happens at the same time in order to keep the gain at 11 dB.)

Example 5 (cont)

For condition 1 (input side), when G_S = 3 dB:

$$g_s = \frac{G_s}{G_{s\max}} = 0.875$$

$$C_s = \frac{g_s S_{11}^*}{1-(1-g_s)|S_{11}|^2} = 0.706 < 120$$

$$R_s = \frac{\sqrt{1-g_s}\left(1-|S_{11}|^2\right)}{1-(1-g_s)|S_{11}|^2} = 0.166$$

Example 5 (cont)

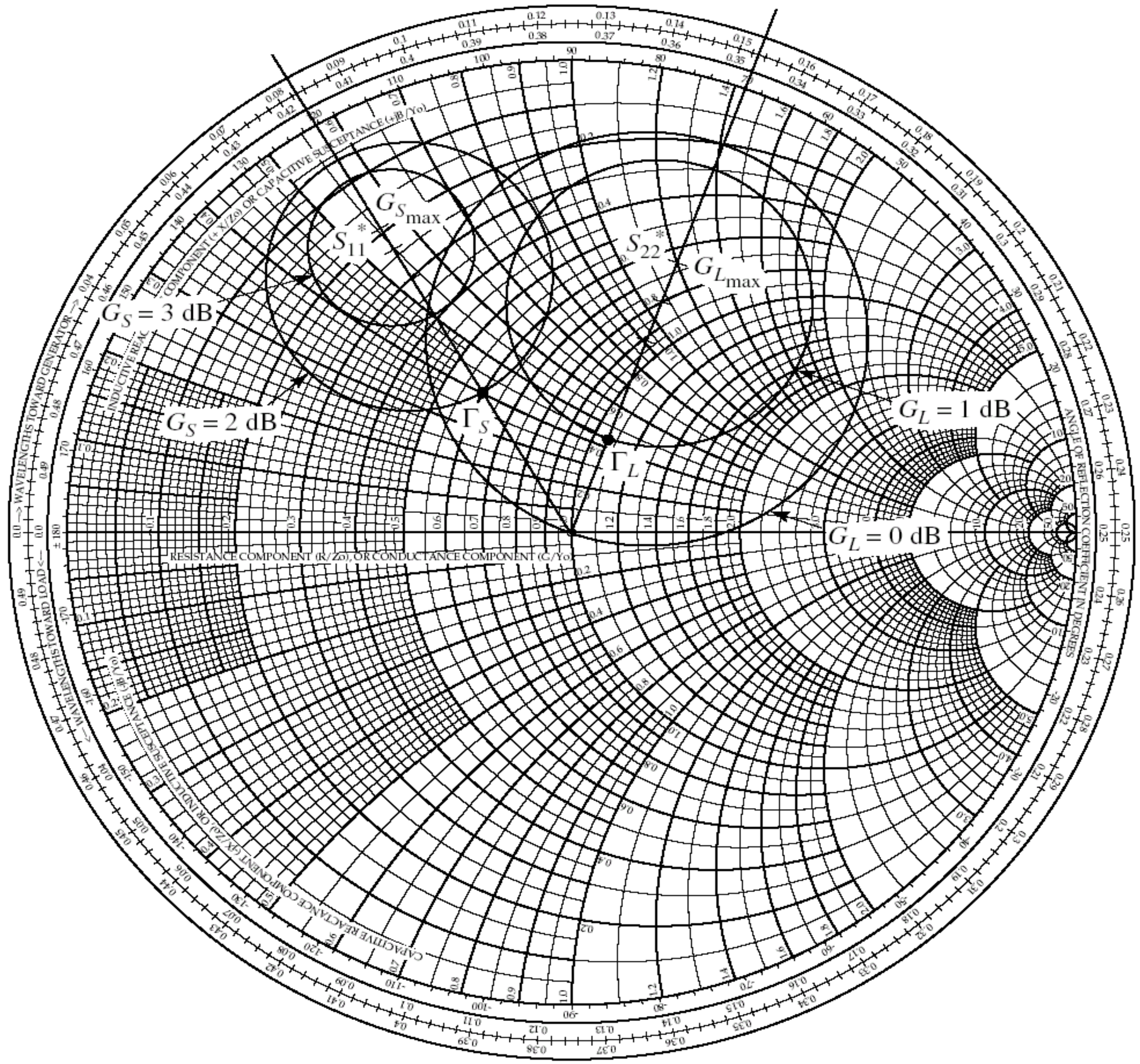
For condition 1 (output side), when G_L = 0 dB:

$$g_L = \frac{G_L}{G_{L\max}} = 0.640$$

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2} = 0.440 < 70$$

$$R_L = \frac{\sqrt{1-g_L}\left(1-|S_{22}|^2\right)}{1-(1-g_L)|S_{22}|^2} = 0.440$$

Example 5 (cont)



(a)

Constant Gain Circles in the Smith Chart

To obtain desired amplifier gain performance

$$\boxed{G_{S\max} = \frac{1}{1-|S_{11}|^2}} \longrightarrow g_S = \frac{G_S}{G_{S\max}} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}(1-|S_{11}|^2)$$

normalize

$$\boxed{G_{L\max} = \frac{1}{1-|S_{22}|^2}} \longrightarrow g_L = \frac{G_L}{G_{L\max}} = \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}(1-|S_{22}|^2)$$

$$\boxed{g_i = \frac{G_i}{G_{i\max}} = \frac{1-|\Gamma_i|^2}{|1-\Gamma_i S_{ii}|^2}(1-|S_{ii}|^2)}$$

This can be written as a circle equation

Circle Equation and Graphical Display

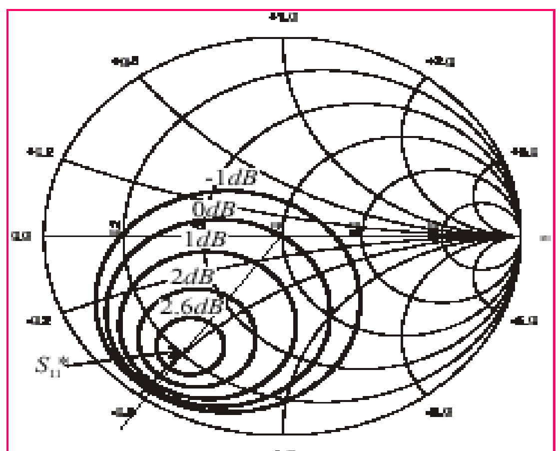
$$(\Gamma_i^R - d_{g_i}^R)^2 + (\Gamma_i^I - d_{g_i}^I)^2 = r_{g_i}^2$$

$$d_{g_i} = \frac{g_i S_{ii}^*}{1 - |S_{ii}|^2 (1 - g_i)}$$

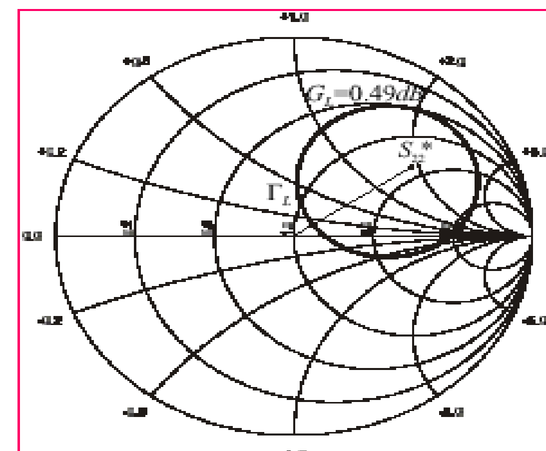
$$r_{g_i} = \frac{\sqrt{1 - g_i}\,(1 - |S_{ii}|^2)}{1 - |S_{ii}|^2 (1 - g_i)}$$

Constant source gain circles

See Ex. 9.7



Constant load gain circle

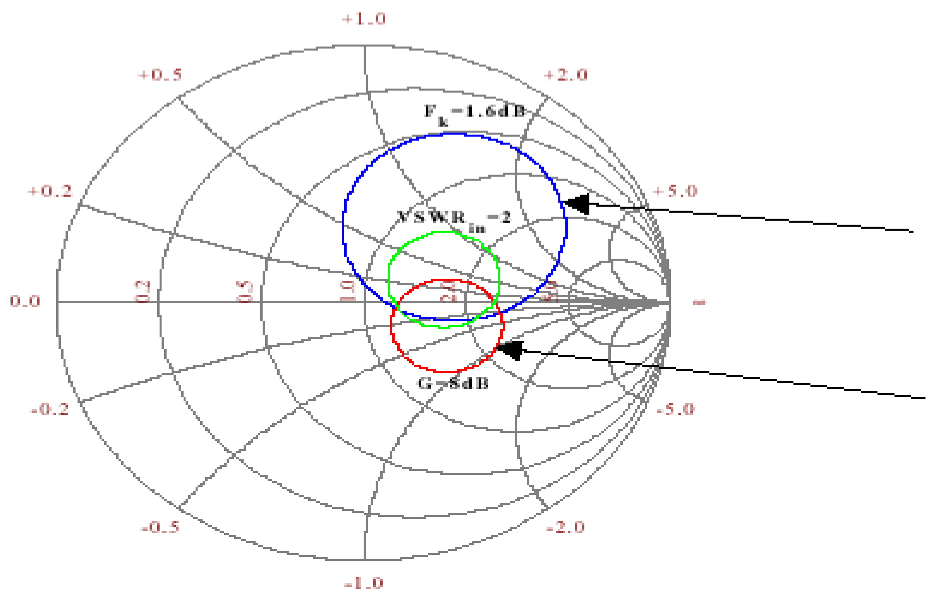


Gain Circles

- Max gain Γ_{imax} =1/(1-$|S_{ii}|^2$) when $\Gamma_i = S_{ii}^*$; gain circle center is at $d_{gi} = S_{ii}^*$ and radius $r_{gi} = 0$
- Constant gain circles have centers on a line connecting origin to S_{ii}^*
- For special case $\Gamma_i = 0$, $g_i = 1 - |S_{ii}|^2$ and

  $d_{gi} = r_{gi} = |S_{ii}|/(1+|S_{ii}|^2)$ implying $\Gamma_i = 1$ (0 dB) circle always passes through origin of Γ_i plane

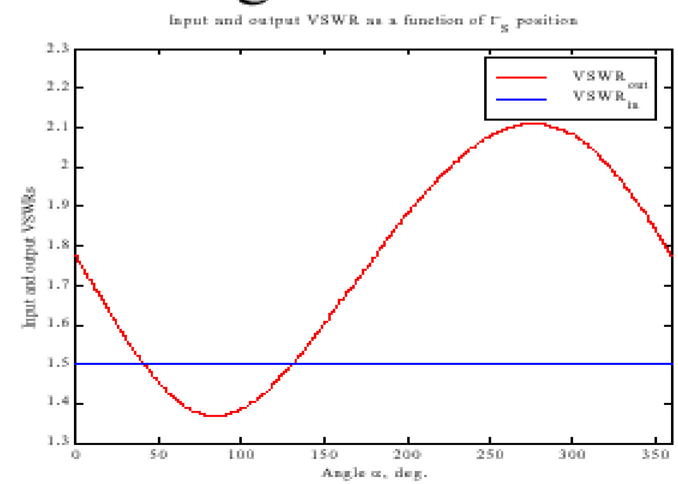
Trade-off Between Gain and Noise



Maximum gain and minimum noise figure are mutually exclusive

Noise figure

Constant gain



Example: Unilateral Constant Gain Circles

- The s-parameters of a bipolar transistor at 500MHz are the following

S11	S21	S12	S22
0.706 -150deg	3.162 80deg	0.01 45deg	0.866 -60deg

- Compute the coordinates of the lateral consant-gain circles of the source, from G_{Smax} to 1dB, in 1 dBsteps, and plot on the Smith Chart.
- G_{Smax}= $\frac{1}{1-|S_{11}|^2}$= $\frac{1}{1-|0.706|^2}$=1.995, 10log(1.995)=3dB
- Next compute g_S, C_S, r_S
- $g_S = \frac{G_S}{G_{Smax}} = G_S\ (1 - |S_{11}|^2)$=1.995$(1 - |0.706|^2)$=1 ; G_S (in dB): 3, 2,1,0,-1
- $\Gamma_S = S_{11}^*$

$$g_S = \frac{G_S}{G_{S\max}} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}(1-|S_{11}|^2)$$

$$C_S = \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}$$

$$R_S = \frac{\sqrt{1-g_S}\left(1-|S_{11}|^2\right)}{1-(1-g_S)|S_{11}|^2}$$

The Corresponding Unilateral Power Gain, with S_{12}=0

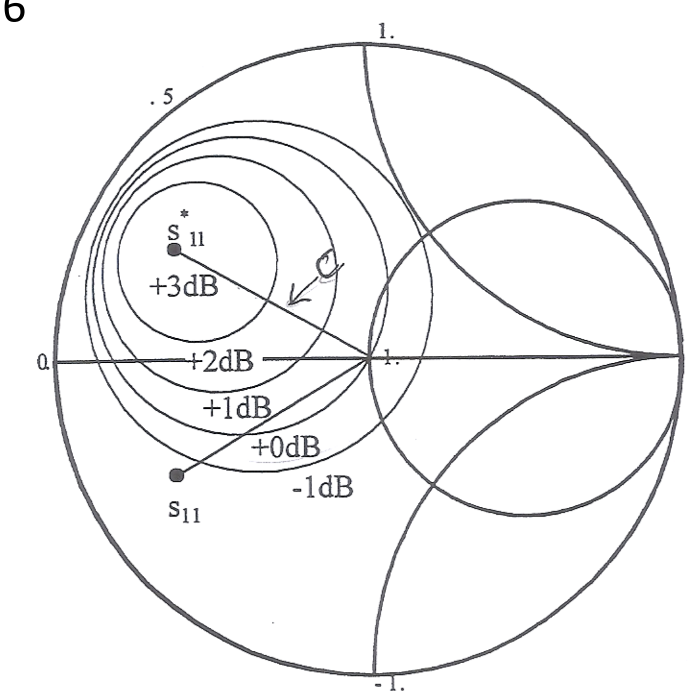
- If $S_{12} = 0$, the maximum gain of a two-port is achieved when $\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$
- The resultant gain change is, $G_{Smax} = \frac{1}{1-|S_{11}|^2}$ and $G_{Lmax} = \frac{1}{1-|S_{22}|^2}$
- The maximum unilateral gain,

$$G_{TUmax}=G_{Smax}G_{omax}G_{Lmax}=\frac{1}{1-|S_{11}|^2}\;|S_{21}|^2\;\frac{1}{1-|S_{22}|^2}$$

$$G_{TUmaxdB}=G_{SmaxdB}+G_{omaxdB}+G_{LmaxdB}$$

Example: Continue

Gs (in dB)	**3**	**2**	**1**	**0**	**-1**
Gs(in factor)	2	1.58	1.26	1	0.79
g_s	1	0.79	0.63	0.50	0.4
$\lvert C_s \rvert$	0.706	0.63	0.55	0.47	0.4
r_1	0	0.25	0.37	0.47	0.56



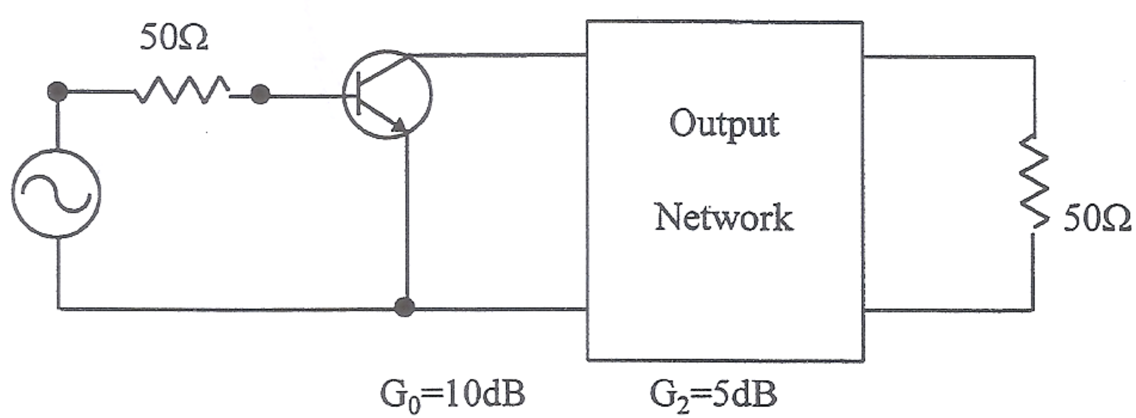
Example: Continue

Design and amplifier with 15dB gain, using the transistor specified on the previous pages. Assume unilateral condition and find the appropriate input or output network(s).

1. The basic transducer gain, s_{21}=3.16=10dB. Therefore +5dB increase is needed.
2. $G_{1\,max} = \dfrac{1}{1-|0.706|^2} = 1.995 \Rightarrow 3\text{dB}$ $\qquad G_{2\,max} = \dfrac{1}{1-|0.866|^2} = 4 \Rightarrow 6\text{dB}$

   Conjugate matching the input would only increase the gain by 3dB.
   Achieving a **partial** match at the output can provide +5dB gain increase.

3. Plot the +5dB unilateral constant gain circle and select the appropriate impedance transforming network (see next page).
4. Cascade the output network to the device and compute the total gain.



Example: Continue

Computation the +5dB load constant-gain circle:

1. Convert +5dB to factor $G_2 = \log^{-1}\dfrac{G_{2dB}}{10} = \log^{-1}(0.5) = 3.16$

$$g_2 = G_2(1-|s_{22}|^2) = 3.16(1-0.866^2) = 0.79$$

2. Compute the center vector

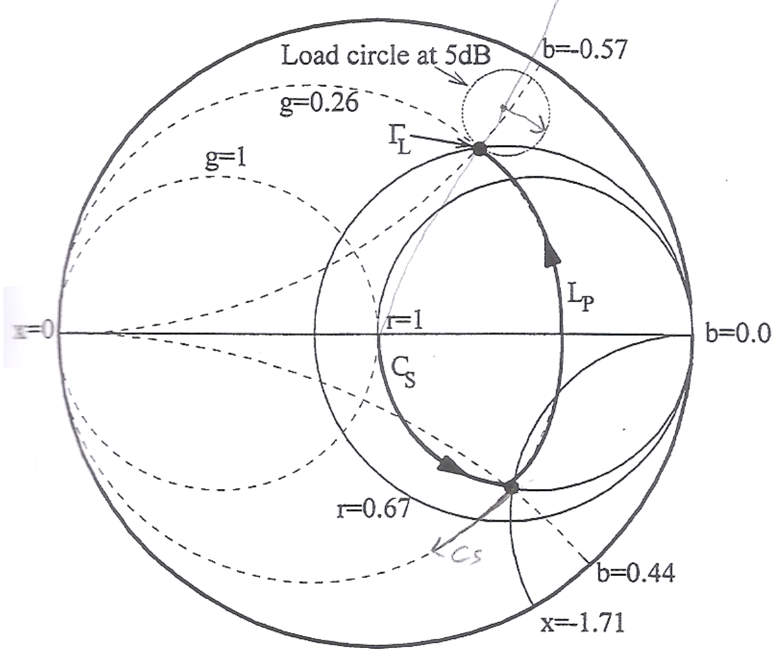
$$C_2 = \frac{g_2 s_{22}^*}{1-|s_{22}|^2(1-g_2)} = \frac{0.79\cdot(0.866)\cdot(\angle 60°)^*}{1-0.866^2(1-0.79)} = 0.81\angle 60°$$

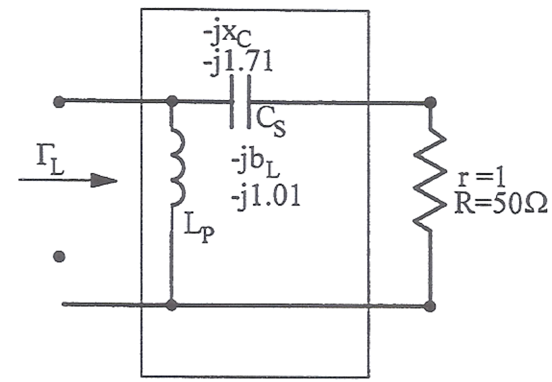
3. Compute the radius

$$r_2 = \frac{\left(1-|s_{22}|^2\right)\sqrt{1-g_2}}{1-|s_{22}|^2(1-g_2)} = \frac{\left(1-0.866^2\right)\sqrt{1-0.79}}{1-0.866^2(1-0.79)} = 0.14$$

Example: Continue

Output Circuit Design at 500MHz





$$L_P = \frac{7.96}{f_{GHz} b_L} = \frac{7.96}{(0.5)(1.01)} = 15.76\text{nH}$$

seçilir.

3.6 – 3.9 pF

$$C_S = \frac{3.18}{f_{GHz} x_C} = \frac{3.18}{(0.5)(1.71)} = 3.72\text{pF}$$

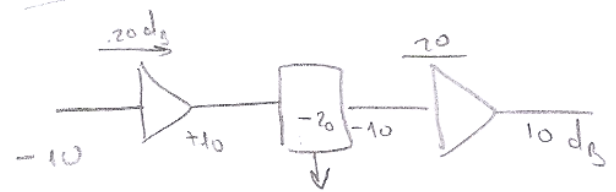
normalized capacitor. impedance.

Example: Finish

$$G_T = |s_{21}|^2 \frac{1-|\Gamma_L|^2}{|1 - s_{22}\Gamma_L|^2} = 3.162^2 \frac{1-0.67^2}{|1-(.866\angle-60°)(0.67\angle 61°)|^2} = 31.62$$

Γ_L seçilir.

$$G_{TdB} = 10\log(31.62) = 15\text{dB}$$



distorsion problemini önlemek için.

Γ_L seçimi: high Q: critical, expensive

bu yüzden Γ_L aşağıda seçilir. ayrıca transistor kararlılığına dikkat edilmeli

Simultaneous Conjugate Match Design

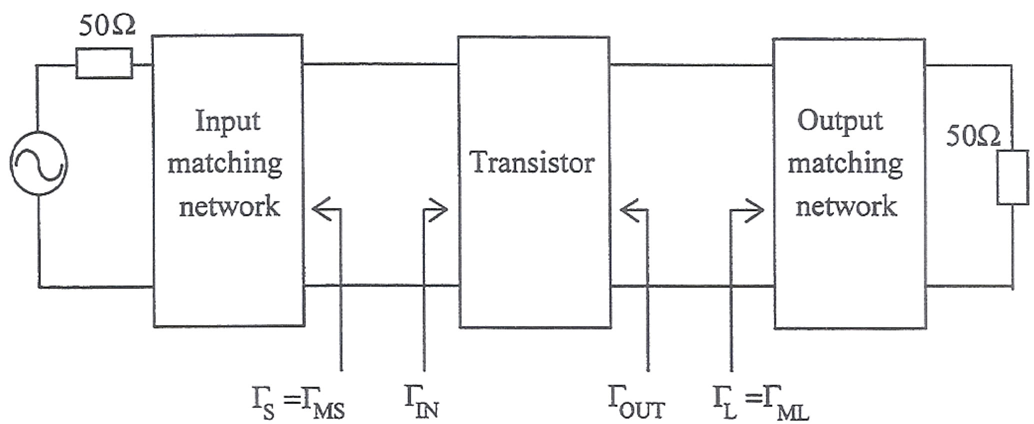
Simultaneous Conjugate Match Source and Load Terminations

When $s_{12} \neq 0$, and the unilateral assumption cannot be made, the input and output reflection coefficients are functions of the s-parameters and the termination at the adjacent port. The conditions required to obtain maximum transducer power gain are

$$\Gamma_{MS} = \Gamma_{IN}^* \qquad \text{and} \qquad \Gamma_{ML} = \Gamma_{OUT}^*$$

If the above conditions occur simultaneously, the new input output impedances of the devices are:

$$\Gamma_{IN} = s_{11} + \frac{s_{12}s_{21}\Gamma_{ML}}{1 - s_{22}\Gamma_{ML}} = \Gamma_{MS}^* \qquad \text{and} \qquad \Gamma_{OUT} = s_{22} + \frac{s_{12}s_{21}\Gamma_{MS}}{1 - s_{11}\Gamma_{Ms}} = \Gamma_{ML}^*$$



Solving for Γ_{MS} and Γ_{ML} from the two equations provides:

$$\Gamma_{MS} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \qquad \text{and} \qquad \Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

where

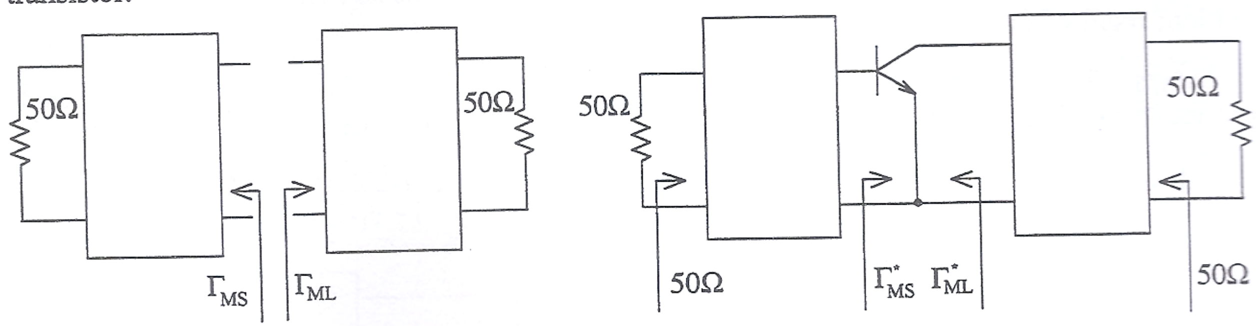
$$B_1 = 1 + |s_{11}|^2 - |s_{22}|^2 - |\Delta|^2 \qquad C_1 = s_{11} - (\Delta)s_{22}^* \qquad |\Delta| = |s_{11}s_{22} - s_{12}s_{21}|$$

$$B_2 = 1 + |s_{22}|^2 - |s_{11}|^2 - |\Delta|^2 \qquad C_2 = s_{22} - (\Delta)s_{11}^*$$

Simultaneous Conjugate Match Maximum Gain

Simultaneous Conjugate Match
Maximum Gain

The matching network may be designed by starting from the 50Ω terminations or from the transistor.



The maximum transducer power gain, under simultaneous conjugate match conditions, is obtained from the previous transducer gain formula, using $\Gamma_S = \Gamma_{MS}$ and $\Gamma_L = \Gamma_{ML}$

$$G_{T,max} = \frac{\left(1-|\Gamma_{MS}|^2\right)|s_{21}|^2\left(1-|\Gamma_{ML}|^2\right)}{\left|(1-s_{11}\Gamma_{MS})(1-s_{22}\Gamma_{ML})-s_{21}s_{12}\Gamma_{MS}\Gamma_{ML}\right|}$$

Substituting for Γ_{MS} and Γ_{ML}, and using the stability factor (K), gives the relation

$$G_{T,max} = \frac{|s_{21}|}{|s_{12}|}\left(K-\sqrt{K^2-1}\right)$$

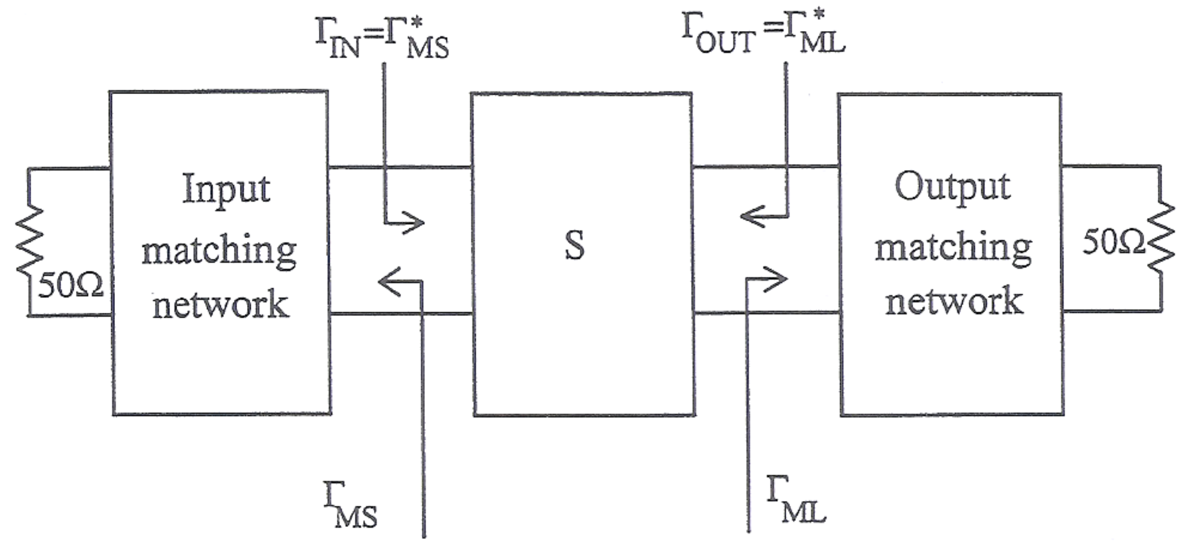
The maximum stable gain is as defined the value of $G_{T,max}$ when K=1, namely

$$G_{MSG} = \frac{|s_{21}|}{|s_{12}|}$$

Example:Simultaneous Conjugate Match Design

Find the input and the output networks for the simultaneous conjugate match at 200MHz, for a transistor with the following s-parameters. Use 50Ω source and load terminations, and compute the matched gain.

$s_{11}=0.4\underline{/162°}$
$s_{12}=0.04\underline{/60°}$
$s_{21}=5.2\underline{/63°}$
$s_{22}=0.35\underline{/-39°}$



Solution

First see if the transistor is stable at the operating frequency and bias point:

$$K = \frac{1+(0.068)^2-(0.4)^2-(0.35)^2}{2(5.2)(0.04)} = 1.74$$

since K is greater than 1, calculate B_1

$B_1=1+(0.4)^2-(0.35)^2-(0.068)^2=1.03$

The transistor passes both tests, $(K>1,\ B_1>0)$, therefore at 200MHz it is unconditionally stable.

Example: Continue

Simultaneous Conjugate Match
Source and Load Calculation

The maximum gain is

$$G_{max} = \left|\frac{S_{21}}{S_{12}}\right|\left(K - \sqrt{K^2 - 1}\right)$$

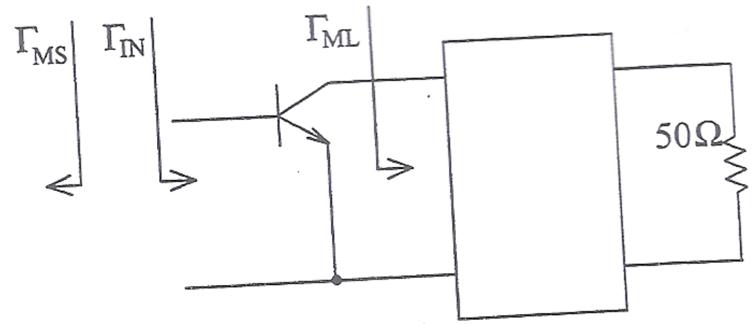
$$= 10\log\frac{5.2}{0.04} + 10\log\left|1.74 - \sqrt{(1.74)^2 - 1}\right|$$

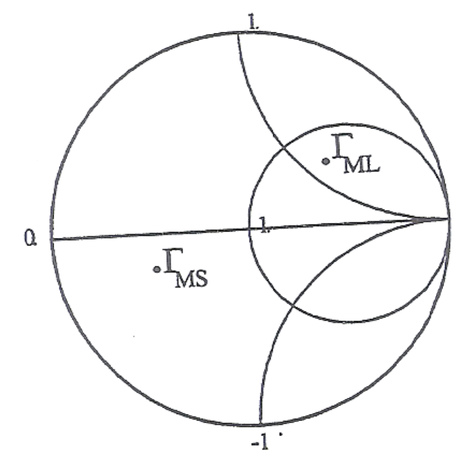
$$= 21.14 + (-5) = 16.1\text{dB}$$

The magnitude of the load matched source and reflection-coefficients can now be found using the expressions from page 3. 2-1:

$$\Gamma_{ML} = \frac{0.958 - \sqrt{(0.958)^2 - 4(0.377)^2}}{2(0.377)\angle -39^\circ} = 0.487\angle 39^\circ$$

$$\Gamma_{MS} = \frac{1.03 - \sqrt{1.03^2 - 4(0.424)^2}}{2(0.424)\angle 162^\circ} = 0.522\angle -162^\circ$$



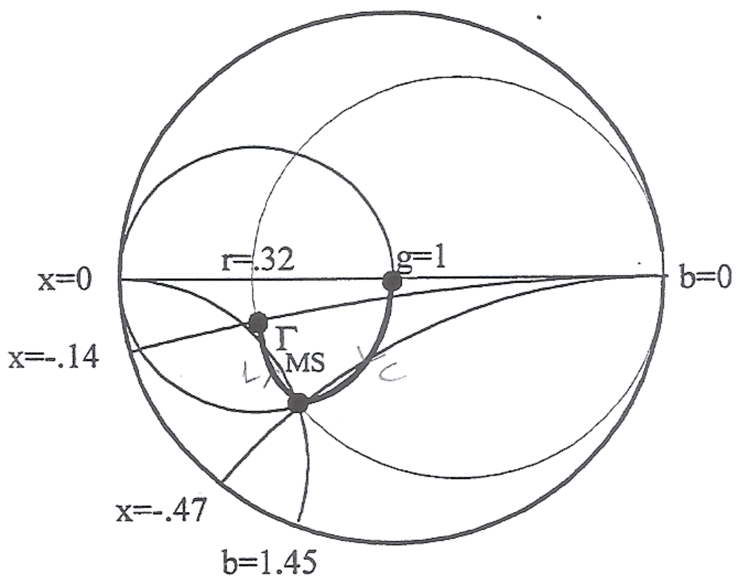


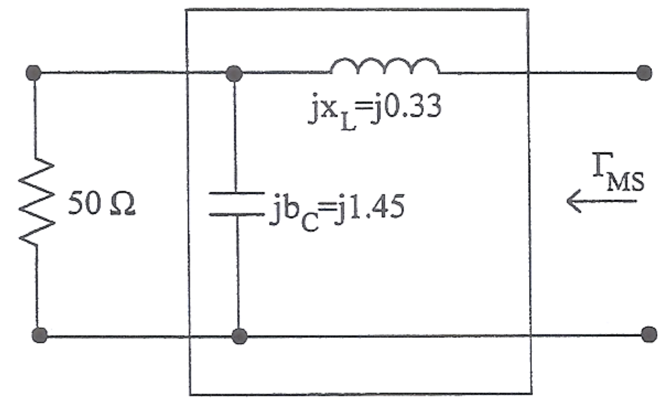
Example: Continue

The desired source reflection coefficient, $\Gamma_{MS} = 0.522\angle -162°$.
Proceeding from the 50Ω source, the input matching network is:

Shunt $C = j1.45 = jb_C$

Series $L = j0.33 = jx_L$





$$C = \frac{3.183(1.45)}{0.2} = 23\text{pF}$$

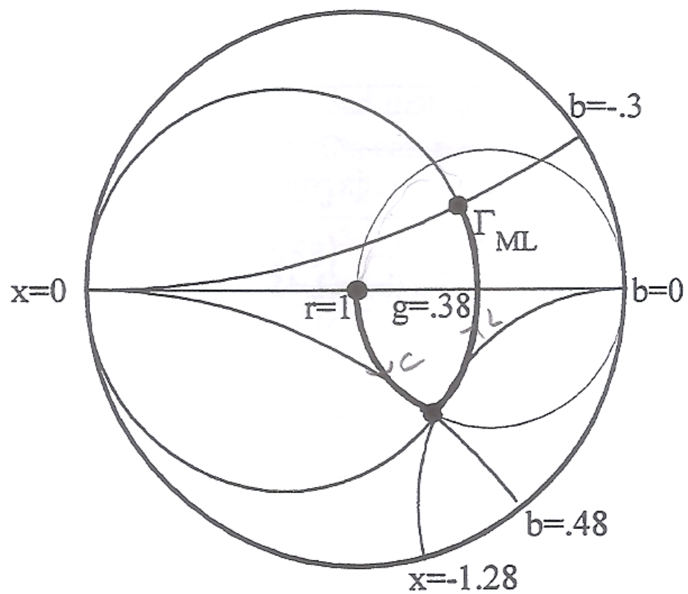
$$L = \frac{7.96(0.33)}{0.2} = 13\text{nH}$$

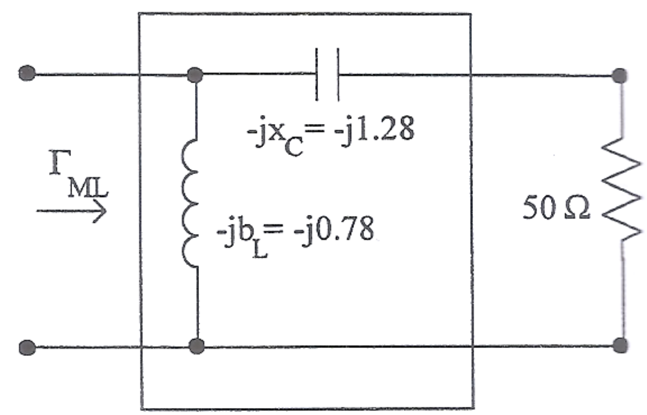
Example: Continue

The computed load reflection coefficient, $\Gamma_{ML} = 0.48\angle 39°$.
The matching output network is designed, proceeding from the 50Ω load:

Series $C = -j1.28 = -jx_C$

Shunt $L = -j0.78 = -jb_L$

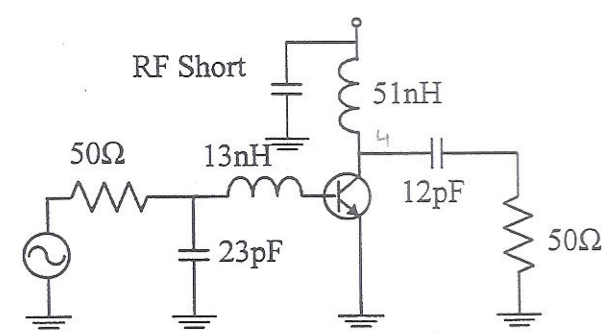




$$C = \frac{3.183}{0.2(1.28)} = 12\text{pF}$$

$$L = \frac{7.96}{0.2(0.78)} = 51\text{nH}$$

RF Circuit Schematics



LOW NOISE AMPLIFIER DESIGN

Low noise amplifier design

- Besides stability and gain, another important design consideration for an RF or microwave amplifier is its noise figure.
- In receiver applications especially, it is often required to have a preamplifier with as low a noise figure as possible since, the first stage of a receiver front end usually has the dominant effect on the noise performance of the overall system.
- Generally it is not possible to obtain both minimum noise figure and maximum gain for an amplifier, so some sort of compromise must be made.
- This can be done by using constant gain circles and circles of constant noise figure to select a usable trade-off between noise figure and gain.

Low noise amplifier (LNA) design

the noise figure of a two-port amplifier can be expressed as

$$F = F_{\min} + \frac{R_N}{G_S}|Y_S - Y_{\text{opt}}|^2, \tag{6.56}$$

$Y_S = G_S + jB_S =$ source admittance presented to transistor.

$Y_{\text{opt}} =$ optimum source admittance that results in minimum noise figure.

$F_{\min} =$ minimum noise figure of transistor, obtained when $Y_S = Y_{\text{opt}}$.

$R_N =$ equivalent noise resistance of transistor.

$G_S =$ real part of source admittance.

The quantities $F_{\min}$, Γ_{opt}, and R_N are characteristics of the particular transistor being used, and are called the *noise parameters* of the device; they may be given by the manufacturer, or measured.

Instead of the admittances Y_S and Y_{opt}, we can use the reflection coefficients Γ_S and Γ_{opt}, where

$$Y_S = \frac{1}{Z_0}\frac{1-\Gamma_S}{1+\Gamma_S}, \tag{6.57a}$$

$$Y_{\text{opt}} = \frac{1}{Z_0}\frac{1-\Gamma_{\text{opt}}}{1+\Gamma_{\text{opt}}}. \tag{6.57b}$$

Using (6.57), the quantity $|Y_S - Y_{\text{opt}}|^2$ can be expressed in terms of Γ_S and Γ_{opt}:

$$|Y_S - Y_{\text{opt}}|^2 = \frac{4}{Z_0^2} \frac{|\Gamma_S - \Gamma_{\text{opt}}|^2}{|1 + \Gamma_S|^2 |1 + \Gamma_{\text{opt}}|^2}. \tag{6.58}$$

Also,

$$G_S = \text{Re}\{Y_S\} = \frac{1}{2Z_0}\left(\frac{1 - \Gamma_S}{1 + \Gamma_S} + \frac{1 - \Gamma_S^*}{1 + \Gamma_S^*}\right) = \frac{1}{Z_0} \frac{1 - |\Gamma_S|^2}{|1 + \Gamma_S|^2}. \tag{6.59}$$

Using these results in (6.56) gives the noise figure as

$$F = F_{\min} + \frac{4R_N}{Z_0} \frac{|\Gamma_S - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_S|^2)|1 + \Gamma_{\text{opt}}|^2}. \tag{6.60}$$

For a fixed noise figure, F, we can show that this result defines a circle in the Γ_S plane.

First, define the *noise figure parameter*, N, as

$$N = \frac{|\Gamma_S - \Gamma_{\text{opt}}|^2}{1 - |\Gamma_S|^2} = \frac{F - F_{\min}}{4R_N/Z_0}|1 + \Gamma_{\text{opt}}|^2, \tag{6.61}$$

which is a constant, for a given noise figure and set of noise parameters. Then rewrite (6.61) as

$$(\Gamma_S - \Gamma_{\text{opt}})(\Gamma_S^* - \Gamma_{\text{opt}}^*) = N(1 - |\Gamma_S|^2),$$

$$\Gamma_S\Gamma_S^* - (\Gamma_S\Gamma_{\text{opt}}^* + \Gamma_S^*\Gamma_{\text{opt}}) + \Gamma_{\text{opt}}\Gamma_{\text{opt}}^* = N - N|\Gamma_S|^2,$$

$$\Gamma_S\Gamma_S^* - \frac{(\Gamma_S\Gamma_{\text{opt}}^* + \Gamma_S^*\Gamma_{\text{opt}})}{N+1} = \frac{N - |\Gamma_{\text{opt}}|^2}{N+1}.$$

Now add $|\Gamma_{\text{opt}}|^2/(N+1)^2$ to both sides to complete the square to obtain

$$\left|\Gamma_S - \frac{\Gamma_{\text{opt}}}{N+1}\right| = \frac{\sqrt{N(N + 1 - |\Gamma_{\text{opt}}|^2)}}{N+1}. \tag{6.62}$$

This expression defines circles of constant noise figure with centers at

$$C_F = \frac{\Gamma_{\text{opt}}}{N+1}, \tag{6.63a}$$

and radii of

$$R_F = \frac{\sqrt{N\left(N + 1 - |\Gamma_{\text{opt}}|^2\right)}}{N+1}. \tag{6.63b}$$

LOW-NOISE AMPLIFIER DESIGN

A GaAs FET is biased for minimum noise figure and has the following S parameters and noise parameters at 4 GHz ($Z_0 = 50\ \Omega$): $S_{11} = 0.60\angle{-60^\circ}$, $S_{21} = 1.9\angle 81^\circ$, $S_{12} = 0.05\angle 26^\circ$, $S_{22} = 0.5\angle{-60^\circ}$; $F_{\min} = 1.6$ dB, $\Gamma_{\text{opt}} = 0.62\angle 100^\circ$, $R_N = 20\ \Omega$. For design purposes, assume the device is unilateral, and calculate the maximum error in G_T resulting from this assumption. Then design an amplifier having a 2.0 dB noise figure with the maximum gain that is possible with this noise figure.

Solution

We first compute the unilateral figure of merit from (6.49):

$$U = \frac{|S_{11}||S_{12}||S_{21}||S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} = 0.059.$$

Then from (6.48) the ratio G_T/G_{TU} is bounded as

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2},$$

or

$$0.891 < \frac{G_T}{G_{TU}} < 1.130.$$

In dB,

$$-0.50 < G_T - G_{TU} < 0.53 \text{ dB},$$

where G_T and G_{TU} are now in dB. Thus, we should expect less than about ± 0.5 dB error in gain due to the approximation of a unilateral device.

Next, we use (6.61) and (6.63) to compute the center and radius of the 2 dB noise figure circle:

$$N = \frac{F - F_{min}}{4R_N/Z_0}|1 + \Gamma_{opt}|^2 = \frac{1.58 - 1.445}{4(20/50)}|1 + 0.62\angle 100°|^2 = 0.0986,$$

$$C_F = \frac{\Gamma_{opt}}{N+1} = 0.56\angle 100°,$$

$$R_F = \frac{\sqrt{N(N+1-\Gamma_{opt}|^2)}}{N+1} = 0.24.$$

This noise figure circle is plotted in Figure 6.14a. Minimum noise figure ($F_{min} = 1.6$ dB) occurs for $\Gamma_S = \Gamma_{opt} = 0.62\angle 100°$. Next, we calculate data for several input section constant gain circles. From (6.54) we compute the following data:

G_S (dB)	g_S	C_S	R_S
1.0	0.805	$0.52\angle 60°$	0.300
1.5	0.904	$0.56\angle 60°$	0.205
1.7	0.946	$0.58\angle 60°$	0.150

These circles are also plotted in Figure 6.14a. We see that the $G_S = 1.7$ dB gain circle just intersects the $F = 2$ dB noise figure circle, and that any higher gain will result in a worse noise figure. From the Smith chart the optimum solution is then $\Gamma_S = 0.53\angle 75°$, yielding $G_S = 1.7$ dB and $F = 2.0$ dB.

For the output section we chose $\Gamma_L = S_{22}^* = 0.5\angle 60°$ for a maximum G_L of

$$G_L = \frac{1}{1 - |S_{22}|^2} = 1.33 = 1.25 \text{ dB}.$$

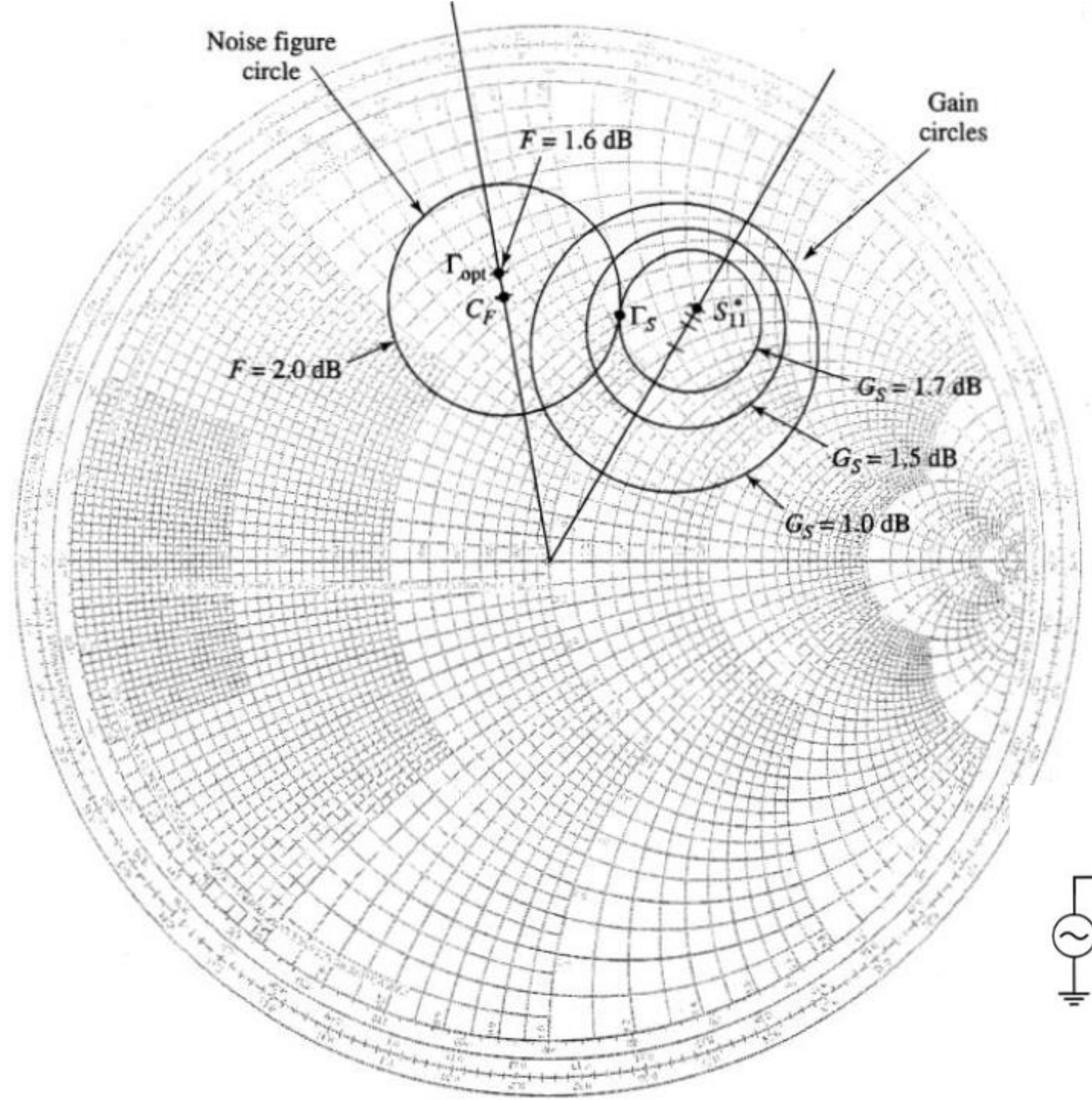
The transistor gain is

$$G_0 = |S_{21}|^2 = 3.61 = 5.58 \text{ dB}.$$

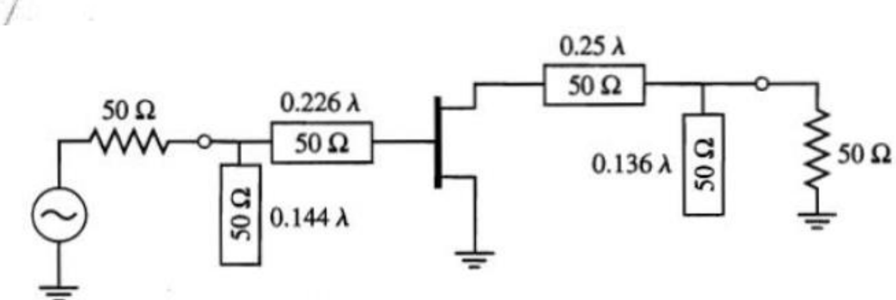
so the overall transducer gain will be

$$G_{TU} = G_S + G_0 + G_L = 8.53 \text{ dB}.$$

The complete AC circuit for the amplifier, using open-circuited shunt stubs in the matching sections, is shown in Figure 6.14b. A computer analysis of the circuit (with $S_{12} \neq 0$) gave a gain of 8.36 dB.



Noise figure circle
F = 1.6 dB
Gain circles
Γopt
CF
ΓS
S11*
F = 2.0 dB
GS = 1.7 dB
GS = 1.5 dB
GS = 1.0 dB



50 Ω
0.226 λ
50 Ω
50 Ω
0.144 λ
0.25 λ
50 Ω
0.136 λ
50 Ω
50 Ω

Example 6

An GaAs FET amplifier is biased for minimum noise figure and has the following S-parameters (Z_0 = 50 Ω):

$$S_{11} = 0.75 < -120$$
$$S_{21} = 2.50 < 80$$
$$S_{12} = 0.00 < 0$$
$$S_{22} = 0.60 < -85$$
$$\Gamma_{opt} = 0.62 < 100$$
$$F_{min} = 1.6 \text{ dB}$$
$$R_N = 20\ \Omega$$

For design purposes, assume the unilateral. Then design an amplifier having 2.0 dB noise figure with the max gain that is compatible with this noise figure.

Example 6 (cont)

Next use the formulas to compute the center and radius of the 2 dB noise figure circle:

$$N = \frac{F - F_{\min}}{4R_N / Z_0} \left|1 - \Gamma_{opt}\right|^2 = 0.0986$$

$$C_F = \frac{\Gamma_{opt}}{N+1} = 0.56 < 100$$

$$R_F = \frac{\sqrt{N\left(N + 1 - \left|\Gamma_{opt}\right|^2\right)}}{N+1} = 0.24$$

The gain of the mismatched transistor is

Example 6 (cont)

The noise figure circle is plotted in the figure. Min noise figure (F_{min} = 1.6 dB) occurs for $\Gamma_S = \Gamma_{opt} = 0.62\angle 100^o$

G_S (dB)	g_S	C_S	R_S
1.0	0.805	$0.52\angle 60^o$	0.300
1.5	0.904	$0.56\angle 60^o$	0.205
1.7	0.946	$0.58\angle 60^o$	0.150

It can be seen that G_S = 1.7 dB gain circle just intersects the F = 2.0 dB noise figure circle, and any higher gain will result in a worse noise figure.

Example 6 (cont)

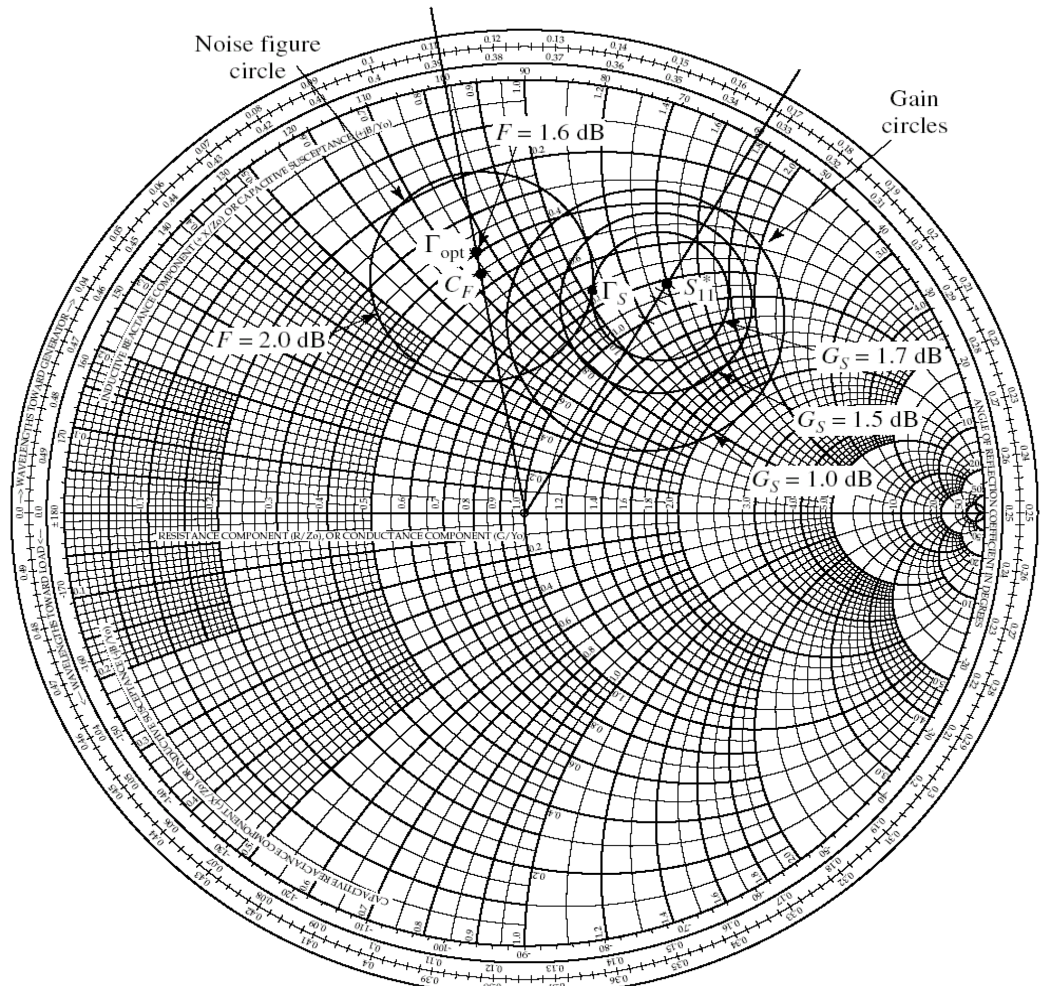
For the output section we choose $\Gamma_L = S_{22}{}^* = 0.5<60^o$ for a max G_L of:

$$G_L = \frac{1}{1-|S_{22}|^2} = 1.33 = 1.25dB$$

$$G_0 = |S_{21}|^2 = 3.61 = 5.58dB$$

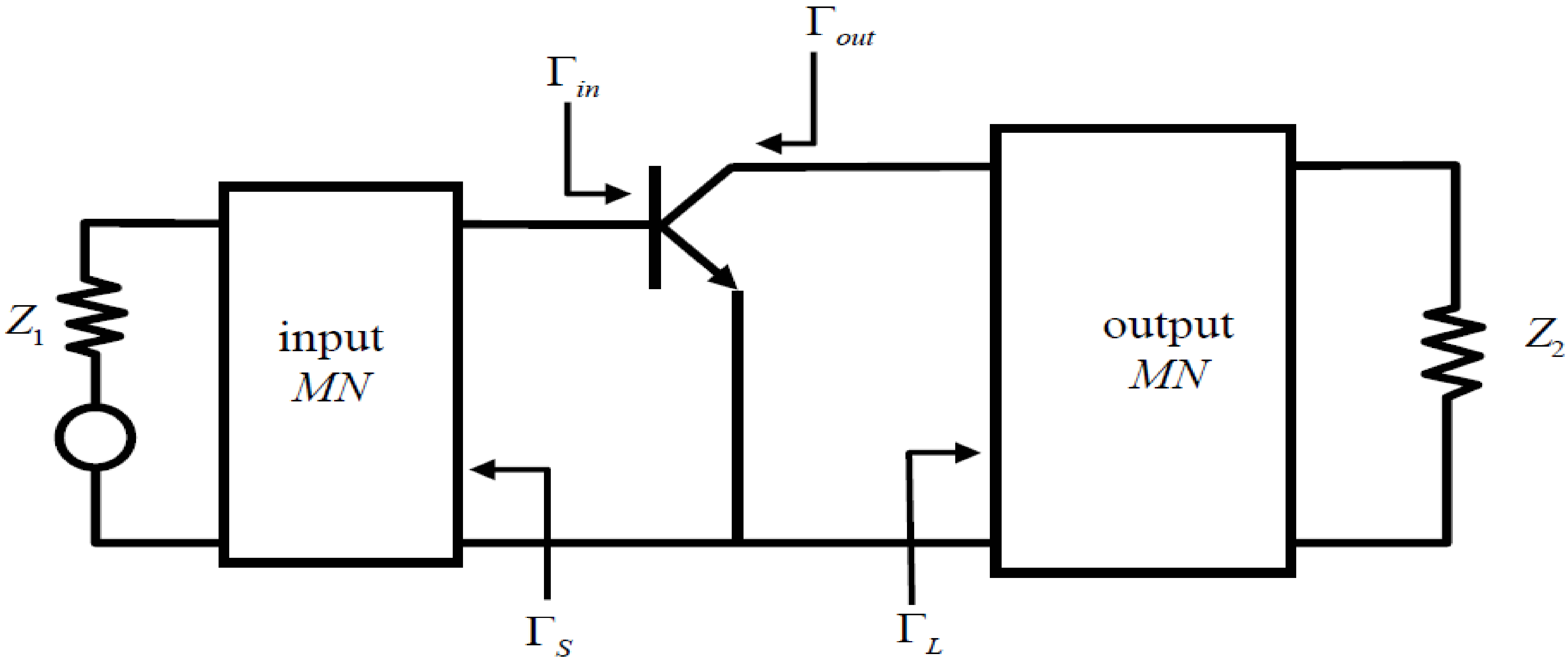
$$G_{TU\max} = G_S + G_0 + G_L = 8.53dB$$

Example 6 (cont)



(a)

Examples


Γ_{out}
Γ_{in}
Z_1
input
MN
output
MN
Z_2
Γ_S
Γ_L

A silicon transitor has following **S**-matrix at 1 GHz with a 50Ω reference impedance,

$$\begin{bmatrix} 0.38\angle{-158^0} & 0.01\angle 34^0 \\ 3.50\angle 80^0 & 0.40\angle{-43^0} \end{bmatrix}$$

The source and load impedances are 25Ω and 40Ω respectively. Compute the unilateral power gain, unilateral available power gain and unilateral transducer power gain.

Solution:

From the **S**-matrix, it is obvious that, the transistor has $S_{12} = 0.01\angle 34^0$ which is comparatively small and closes to zero. Hence, it can be assumed that the transistor is unilateral.

The value of source reflection coefficient Γ_s and load reflection coefficient Γ_l can be calculated as

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0} \quad = \frac{25-50}{25+50} \quad = -\frac{1}{3} \quad = \frac{1}{3}\angle 180^0$$

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l - Z_0} \quad = \frac{40-50}{40+50} \quad = -\frac{1}{9} \quad = \frac{1}{9}\angle 180^0$$

From the generic formula we can calculate the power gain of the transistor,

$$G = \frac{|S_{21}|^2(1 - |\Gamma_l|^2)}{(1 - |\Gamma_{in}|^2)|1 - S_{22}\Gamma_l|^2}$$

Now, as the transistor is approximated as unilateral, therefore, $\Gamma_{out} \approx S_{22}$ and $\Gamma_{in} \approx S_{11}$. So,

$$\begin{aligned} G = G_U &= \frac{|S_{21}|^2(1 - |\Gamma_l|^2)}{(1 - |S_{11}|^2)|1 - S_{22}\Gamma_l|^2} \\ &= \frac{(3.50)^2 \times 0.99}{0.86 \times [1 - 0.05\angle 137^0]^2} \\ &= \frac{3.5^2 \times 0.99}{0.86 \times 1.08} \\ &= 13.05 \\ &= 11.16dB \end{aligned}$$

Similarly by applying the generic formula of available power gain as given below, the required parameter can be calculated.

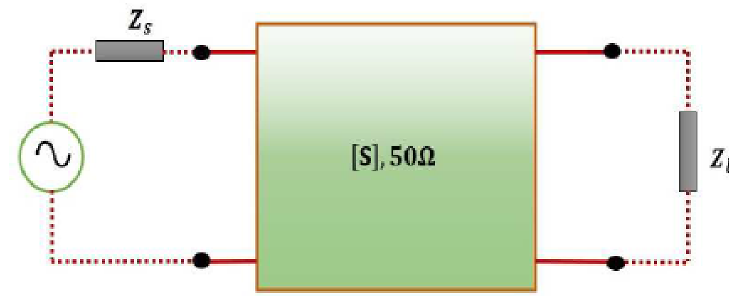
$$G_A = G_{AU} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2(1-|S_{22}|^2)}$$
$$= \frac{3.5^2 \times 0.89}{0.84 \times |1-0.13\angle 27^0|^2}$$
$$= \frac{3.5^2 \times 0.89}{0.84 \times 0.79} = 16.49 = 12.15dB$$

In similar fashion,

$$G_T = G_{TU} = \frac{|S_{21}|^2[1-|\Gamma_s|^2][1-\Gamma_l]^2}{|1-S_{11}\Gamma_s|^2|1-S_{22}\Gamma_l|^2}$$
$$= \frac{3.5^2 \times 0.99 \times 0.89}{|1-0.13\angle 27^0|^2|1-0.05\angle 137^0|^2}$$
$$= \frac{3.5^2 \times 0.99 \times 0.89}{0.79 \times 1.08} = 12.65 = 11.02dB$$

A transistor has following S matrix at 1 GHz (50Ω) and connected to the network given below.

$$\begin{bmatrix} 0.91\angle{-44^0} & 0.06\angle 68^0 \\ 3.92\angle 149^0 & 0.93\angle{-17^0} \end{bmatrix}$$



Determine the value of Γ_s and Γ_l to maximize the transducer power gain.

Solution:

We need to determine the values Γ_s and Γ_l such that the transducer gain is maximum. Therefore, at first we need to check the stability of the transistor.

By applying the Rollet's stability criteria,

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\triangle|^2}{2|S_{12}S_{21}|}$$

where $\triangle = S_{11}S_{22} - S_{12}S_{21}$ and $|\triangle| = |0.91\angle{-44^0}0.93\angle{-17^0} - 3.92\angle 149^0 0.06\angle 68^0| = |0.6 - j0.6| = 0.84 < 1$. Hence,

$$\begin{aligned} K &= \frac{1 - 0.91^2 - 0.93^2 + 0.84^2}{2 \times 3.92 \times 0.06} \\ &= 0.026 \\ &< 1 \end{aligned}$$

As the transistor fails the Rollet's stability test, the transistor network will be conditionally stable for some values of Z_s and Z_l. Therefore, we need to determine the range of values Γ_l and Γ_s for which the transistor network will be stable. We need to determine the centers and radii of input and output stability circles.

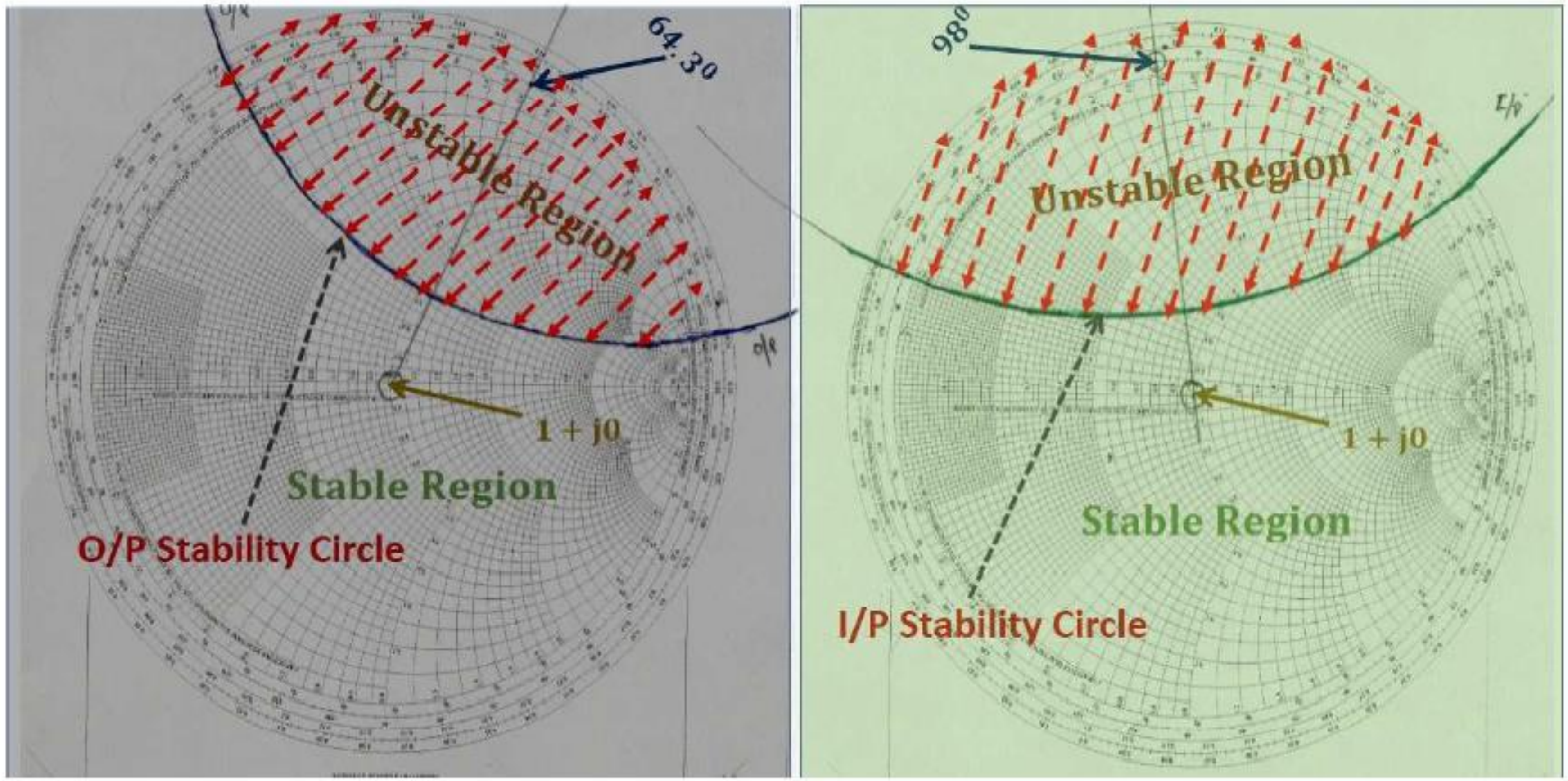


Figure 1: The stable and unstable regions for a transistor with specific S-matrix

$$C_l = \frac{(S_{22} - \triangle S_{11}^*)^*}{|S_{22}^2| - |\triangle|^2}$$

$$= \frac{0.93\angle{-17^0} - (0.84\angle{-45^0} \times 0.91\angle 44^0)}{0.93^2 - 0.84^2} = 0.78 + j1.62 \;\; = 1.8\angle 64.3^0$$

$$R_l = \frac{|S_{12}S_{21}|}{|S_{22}^2| - |\triangle|^2} = \frac{0.235}{0.93^2 - 0.84^2} = 1.48$$

$$C_s = \frac{(S_{11} - \triangle S_{22}^*)^*}{|S_{11}^2| - |\triangle|^2}$$

$$= \frac{0.91\angle{-44^0} - (0.84\angle{-45^0} \times 0.93\angle 17^0)}{0.91^2 - 0.84^2}$$

$$= -0.29 + j2.17 \;\; = 2.19\angle 97.6^0$$

$$R_s = \frac{|S_{12}S_{21}|}{|S_{11}^2| - |\triangle|^2}$$

$$= \frac{0.235}{0.91^2 - 0.84^2} = 1.92$$

The input stability circle divides the smith chart into two parts. The region of stability can be determined by considering the valued of $|S_{11}|$. If $|S_{11}| < 1$, then the region containing the center of the smith chart will be stable region for both input stability circle case (i.e. load reflection coefficient) and output stability circle case (i.e. source reflection coefficient).

Same recipe can be applied for identifying the stable region for output side but the decision has to be taken based on the value $|S_{22}|$. In the present case the $|S_{11}| < 1$ and $|S_{22}| < 1$, thus, the portion of smith chart containing the center will be stable region. The stable and unstable regions are determined using Smith chart and shown in Figure-1.

Now, $\Gamma_s = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$ and $\Gamma_l = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$. Where,

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\triangle|^2 \quad = 0.257$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\triangle|^2 \quad = 0.331$$

$$\begin{aligned} C_1 &= S_{11} - \triangle S_{22}^* \\ &= 0.91\angle{-44^0} - (0.84\angle{-45^0} \times 0.93\angle{17^0}) \\ &= -0.03 - 0.27j \;= 0.27\angle{-96.3^0} \end{aligned}$$

In order to have a solution of Γ_l and Γ_s for conjugate matching, the quantities, $B_1^2 - 4|C_1|^2$ and $B_1^2 - 4|C_1|^2$ must have positive values. In this case, none of the quantities gives a positive value. Therefore, the maximum gain can not be obtained and the transistor network can not be matched to achieve the maximum gain.

An amplifier uses a GaAs HBT device having the following scattering parameters ($Z_0 = 50\Omega$). The input of the transistor is connected to a source with $V_s = 2V$(peak) and $Z_s = 25\Omega$ and the output is connected to $Z_l = 100\Omega$ load. Compute the available power from the source and power delivered to the load.

$$\begin{bmatrix} 0.61\angle{-170^0} & 0.06\angle 70^0 \\ 2.3\angle 80^0 & 0.72\angle{-25^0} \end{bmatrix}$$

Solution:

The reflection coefficients Γ_s and Γ_l can be determined as

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0} = \frac{25-50}{25+50} = -\frac{1}{3} = \frac{1}{3}\angle 180^0$$

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0} = \frac{100-50}{100+50} = \frac{1}{3}$$

From the signal flow graph, the value of input and output reflection coefficients can be determined as

$$\begin{aligned}\Gamma_{in} &= S_{11} + \frac{S_{12}S_{21}\Gamma_l}{1 - S_{22}\Gamma_l} \\ &= 0.61\angle{-170^0} + \frac{2.3\angle 80^0 \times 0.6\angle 70^0 \times \frac{1}{3}}{1 - 0.72\angle{-25} \times \frac{1}{3}} = -0.65 - j0.07 = 0.65\angle{-173.77^0}\end{aligned}$$

$$\begin{aligned}\Gamma_{out} &= S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \\ &= 0.72\angle{-25^0} + \frac{2.3\angle 80^0 \times 0.6\angle 70^0 \times \frac{1}{3}\angle 180^0}{1 - 0.61\angle{-170^0} \times \frac{1}{3}\angle 180^0} = 0.703 - j0.33 = 0.776\angle{-25^0}\end{aligned}$$

Now, the average power supplied by the source to the transistor network is

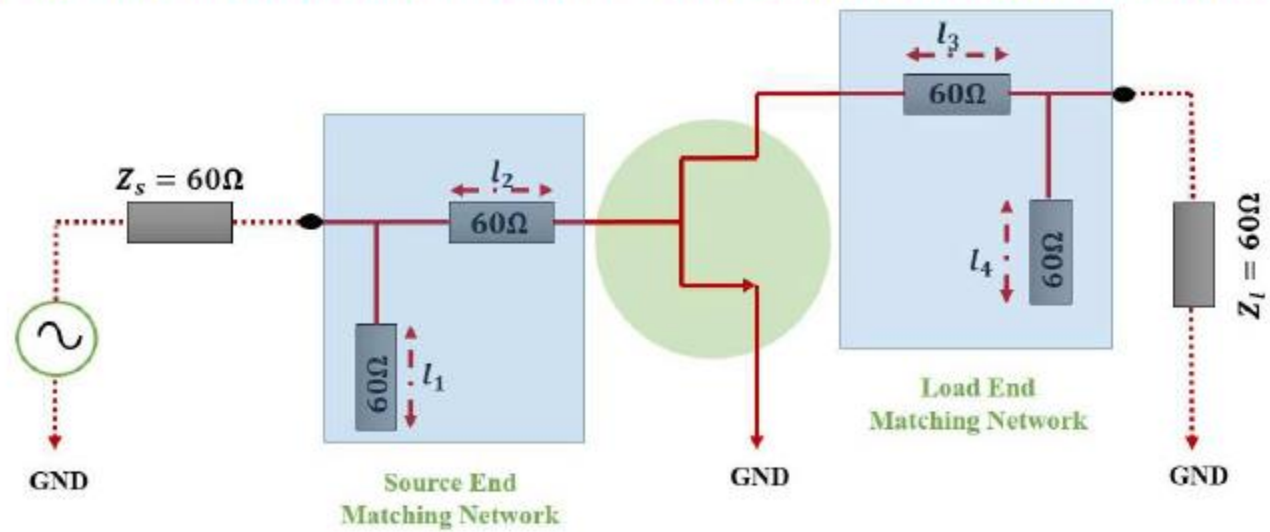
$$
\begin{aligned}
P_s &= \frac{1}{2Z_0}|V^+|^2 \\
&= \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_s|^2}{|-\Gamma_s\Gamma_{in}|^2} \\
&= \frac{2}{8\times 50} \times \frac{|1-\frac{1}{3}\angle 180^0|^2}{|1-(\frac{1}{3}\angle 180^0 \times 0.65\angle{-173^0})|^2} \\
&= 0.014W
\end{aligned}
$$

In similar fashion, the power delivered to the load can be determined as

$$
\begin{aligned}
P_l &= \frac{|V_s|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_l|^2)|1-\Gamma_s|^2}{|1-S_{22}\Gamma_l|^2|1-\Gamma_s\Gamma_{in}|^2} \\
&= \frac{2}{8\times 50} \times \frac{2.3^2 \times (1-\frac{1}{9}) \times |1-\frac{1}{3}\angle 180^0|^2}{|1-0.72\angle{-25^0}|^2 \times |1-(13\angle 180^0 \times 0.65\angle{-173.77^0})|^2} \\
&= 0.108W
\end{aligned}
$$

Determine the length of short-circuited shunt stubs (l_1 and l_4) & the length of the transmission line (l_2 and l_3) at 3 GHz for the network shown below. The s-matrix of the network is.

$$\begin{bmatrix} 0.66\angle{-142^0} & 0.03\angle{62^0} \\ 2.39\angle{76^0} & 0.72\angle{-68^0} \end{bmatrix}$$



Solution:

The transmission lines used in the matching networks are assumed to be lossless. In order to design the matching network, wee need to check the stability conditions of the transistor from the supplied S$-$matrix values.

Applying Rollet's stability criteria we get

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\triangle|^2}{2|S_{12}S_{21}|}$$

where $\triangle = S_{11}S_{22} - S_{12}S_{21}$ and $|\triangle| = |0.66\angle{-142^0}0.72\angle{-68^0} - 2.39\angle 54^0 0.03\angle 62^0| = |-0.38 + j.173| = 0.42 < 1$. Hence,

$$K = \frac{1 - 0.66^2 - 0.72^2 + 0.42^2}{2 \times 2.39 \times 0.03} = 1.55 \quad > 1$$

Therefor, the transistor network passes the Rollet's stability criteria. So, the network is absolutely stable for any values of Γ_s and Γ_l.

Hence, $\Gamma_s = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$ and $\Gamma_l = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$. Where,

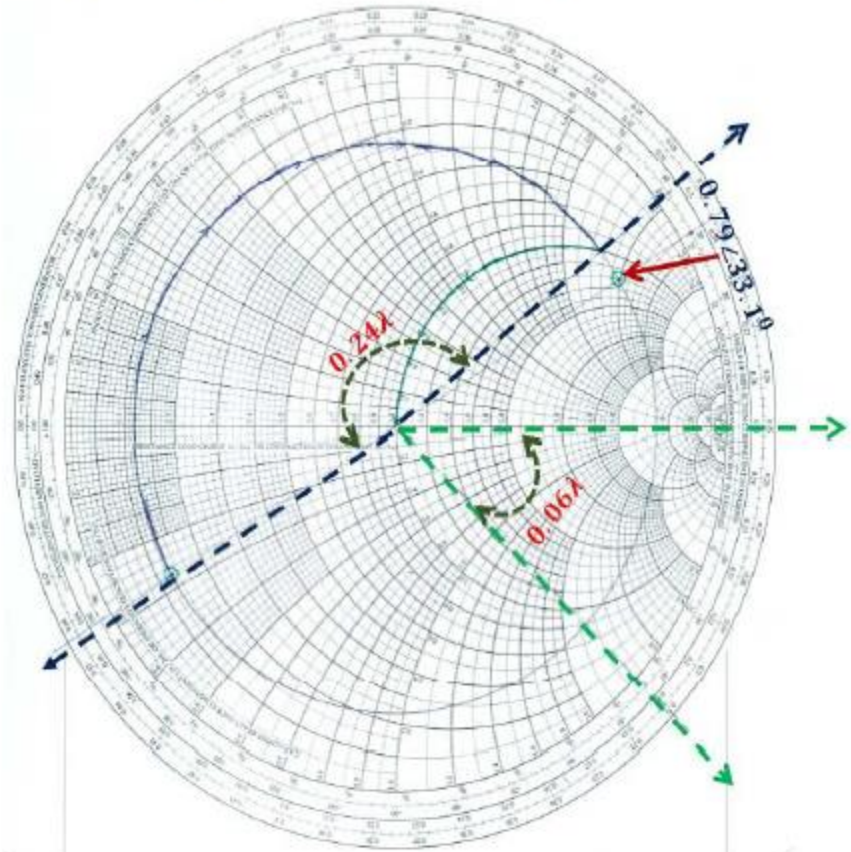
$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\triangle|^2 \quad = 0.74$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\triangle|^2 \quad = 0.91$$

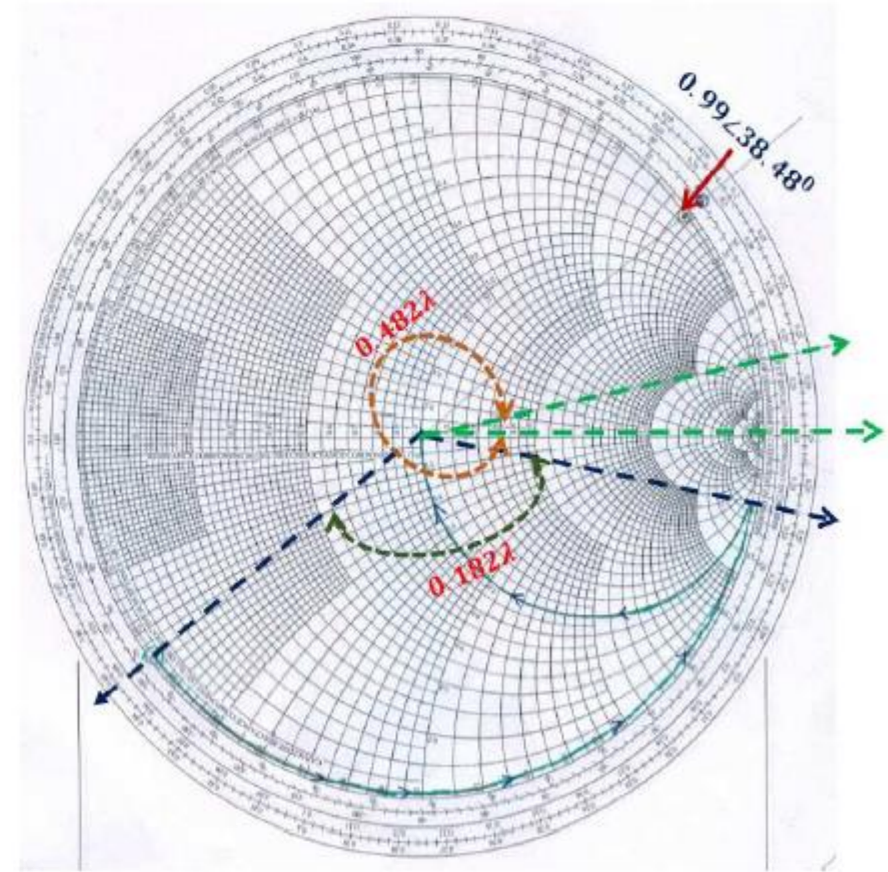
$$C_1 = S_{11} - \triangle S_{22}^* \quad = -0.3 - 0.198j$$

$$C_2 = S_{22} - \triangle S_{11}^* \quad = 1.98 \times 10^{-4} - j0.73$$

So, $\Gamma_s = 0.78 + 0.62j = 0.79\angle 33.1^0$ and $\Gamma_l = 0.78 + 0.62j = 0.99\angle 38.48^0$. The length of the short circuited shunt stubs and the length of the transmission line can be easily determined by using Smith chart. Smith chart based solutions for the same are shown below.



Smith Chart based solution for input matching network



Smith Chart based solution for output matching network

It is mentioned in the problem that all the values are taken at a frequency of 3 GHz. Hence, the value of wavelength $\lambda = \frac{3\times10^{10}}{3\times10^{9}} = 10$ cm.

So, the length of the short circuited stubs $l_1 = 0.06\lambda = 0.6$cm, $l_4 = 0.182\lambda = 1.82$cm, and the length of the lossless transsion lines are $l_2 = 0.24\lambda = 2.4$ cm and $l_3 = 0.482\lambda = 4.82$cm. The entire transistor network after conjugate matching is shown in Figure-2

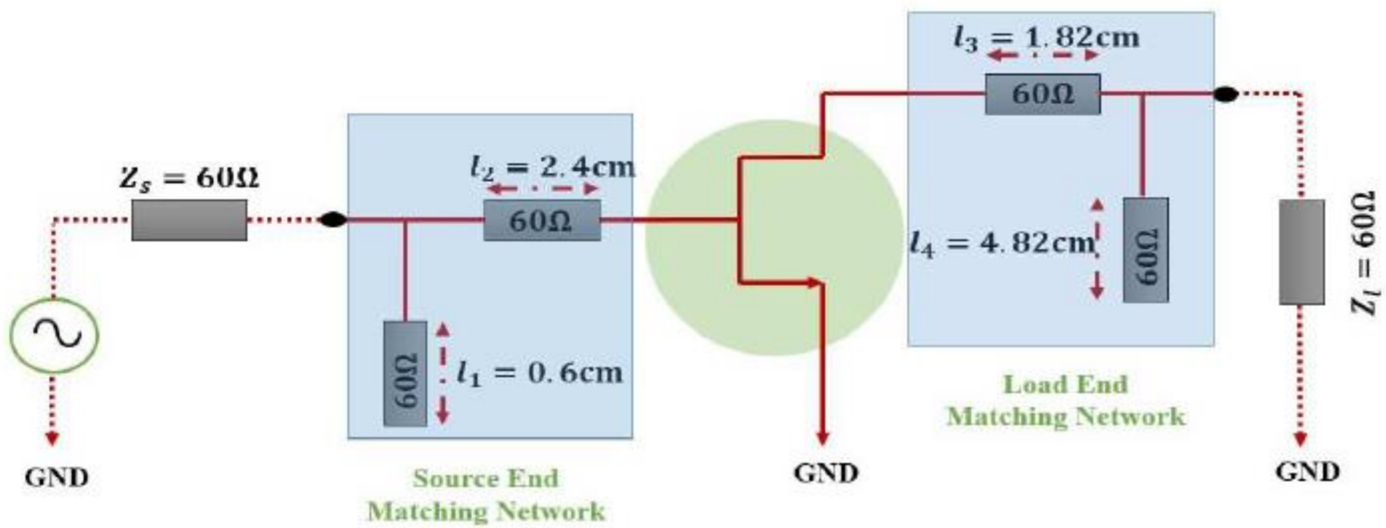


Figure 2: A transistor network with conjugate matching sections

Kaynaklar

- https://www.ece.ucsb.edu/~long/ece145a/ampdesign.pdf
- Amplifiers, Prof. Tzong-Lin Wu. EMC Laboratory. Department of Electrical Engineering. National Taiwan University

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Sincerely,

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